

**Modelling the Nature of Close-out Netting
on Bank Portfolios**

ALDO TARANTO

MASTER OF BUSINESS

2006

RMIT University

Modelling the Nature of Close-out Netting on Bank Portfolios

A thesis submitted in fulfillment of the requirements for the degree of
Master of Business in Accounting

ALDO TARANTO
BSc, GradDipEd, MBSys

School of Accounting and Law
RMIT Business
RMIT University

10-2006

Abstract

The stochastic volatility of daily foreign exchange (FX) derivatives poses a number of risks for the international banking community. Settlement risk, liquidity risk and capital adequacy are just a few immediate concerns that arise from such volatility. This thesis examines the impact of close-out netting on minimising the stochastic volatility of inter-bank FX derivatives. The problem with close-out netting is that although it is a simple formula of taking the differences between two banks at one point in time, it is the stochastic and volatile nature of FX rates that makes measuring the full impact of netting difficult.

The objective of this thesis is to establish a realistic international banking framework or modelling environment in which close-out netting can be scientifically applied and examined. Five international daily FX rates will be used as sufficient approximations for five international banks. A generalised autoregressive conditionally heteroschedastic (GARCH) modelling approach is adopted as a robust and rich FX volatility paradigm. Then through Monte Carlo simulation of the resulting fitted GARCH models, we generate the distributions -with and without close-out netting. Through statistical techniques, we estimate the impact FX volatility due to close-out netting.

The findings of this thesis are interesting, showing that close-out netting is far more than just a simple mathematical process. Netting surely does reduce each bank's exposure to FX volatility, however, its multivariate nature reveals some important results for banking risk research and bank analysts. These include; (1) Some banks have little or no impact due to close-out netting (2) one "minimally leveraged" bank may reduce their exposure down to zero (3) some banks may be worse off under netting due to there being a 'risk transfer' by certain banks (4) some of the resulting distributions are non-standard, highlighting the underlying mechanism of this multivariate process. These results answer many questions about close-out netting and raise new questions for further research that is beyond the scope of this thesis.

Declaration

I certify that except where due acknowledgement has been made, the work is that of the author alone; the work has not been submitted previously, in whole or in part, to qualify for any other academic award; the content of the thesis is the result of work which has been carried out since the official commencement date of the approved research program; and, any editorial work, paid or unpaid, carried out by a third party is acknowledged.

Aldo Taranto

30/10/2006

Acknowledgments

I would like to take this opportunity to cite and thank the following two main colleagues.

- Dr. Chris White -for broadening my way of thinking.
- Prof. Clive Morley -for coordinating the success of this research.

Finally, I would like to thank my wife Tania for ensuring that I took a balanced dedication to this research.

Contents

Abstract	iii
Declaration	iv
Acknowledgments	v
List of Figures	viii
List of Tables	1
Chapter 1 Introduction	1
1.1 Close-out Netting	3
1.2 Other Definitions	5
1.3 Thesis Structure	6
Chapter 2 Literature Review	8
2.1 Institutional and Historical Framework of Netting	8
2.2 Netting Legislation	11
2.3 Credit Risk Models	14
2.4 Netting Research	16
Chapter 3 Modelling Methodology	20
3.1 Data Preparation	21
3.2 Stochastic Volatility of FX Rates Methodology	22
3.3 GARCH Methodology	23
3.4 Netting Methodology	26
3.4.1 Generalised Netting Model	26
3.4.2 Example of Two Banks	31
3.5 MC Simulation Methodology	32

3.5.1	MC Expected Error and Estimates	34
3.5.2	Extensions of MC Simulation for Financial Time Series Data	35
Chapter 4	Modelling Results	37
4.1	FX Rate Time Series Data	37
4.2	GARCH Models	55
4.3	MC GARCH Simulations	79
4.4	Application of Netting Model	90
Chapter 5	Conclusions	105
5.1	Concluding the Results	105
5.2	Addressing Possible Accounting Issues	110
5.3	Further Research	111
5.3.1	Determining the Relationship Between Close-out Netting and Capital Adequacy	112
5.3.2	Collateralisation Under Uncertainty	112
5.3.3	Incrementally Increasing the Number of Counterparties	112
5.3.4	Varying Portfolio Asset Group Concentrations	113
5.3.5	Analysing Stages of Economic Cycles	113
5.3.6	Enhancing Close-out Netting Research via Copulas	113
5.3.7	Generating Quasi-Random Numbers for Quasi-Monte Carlo Simulation	114
Bibliography		116
Appendix A	Additional Theoretical Results	124
A.1	Example of Three Banks	124
A.2	Illustration of Four and Five Banks	127
A.3	Vector-Sequence Notation	128
A.4	Bank's Total Exposure	128
A.5	Netting System's Total Exposure	131
Appendix B	VaR	132
Appendix C	Further Quasi-Random Number Details	135

List of Figures

3.1	Use of Two Years of Daily FX Rate Data	22
3.2	Exposures Between Two Banks	31
4.1	Daily FX Rate Data for 2004-2005	38
4.2	Daily FX Rate Data for 2004	40
4.3	Daily FX Rate Differences for 2004-2005	42
4.4	Daily FX Rate Differences for 2004	43
4.5	FX Daily Differences and Two Conditional Standard Deviations Superimposed for 2004	45
4.6	ACF of FX Rate Daily Differences for 2004	47
4.7	ACF of FX Rate Squared Daily Differences for 2004	48
4.8	FX Rate Squared Daily Differences Cross Correlation with Daily Differences for 2004	50
4.9	FX Rate Differences Histograms for 2004	52
4.10	FX Rate Differences QQ-Plots for 2004	54
4.11	USD/EUR FX Daily Differences and GARCH(1,1) Conditional Standard Devia- tion for 2004 Using Student-t Distribution	56
4.12	FX Daily Differences and GARCH(1,1) Conditional Standard Deviation for 2004	58
4.13	USD/EUR FX Daily Differences and GARCH(1,1) Conditional Standard Devia- tion for 2004 Using Student-t Distribution	59
4.14	FX GARCH(1,1) Residuals for 2004	60
4.15	FX GARCH(1,1) Standardised Residuals for 2004	62
4.16	ACF of Standardised FX GARCH(1,1) Residuals for 2004	64
4.17	ACF of Squared Standardised FX GARCH(1,1) Residuals for 2004	66
4.18	FX GARCH(1,1) Squared Residuals Cross Correlation with Residuals for 2004 .	68
4.19	FX Histograms of Standardised GARCH(1,1) Residuals for 2004	70

4.20	FX QQ-Plot of Standardised GARCH(1,1) Residuals for 2004	72
4.21	250 MC Simulations of FX GARCH(1,1) Models for 2005 Using Circle Markers .	80
4.22	250 MC Simulations of FX GARCH(1,1) Models for 2005 Using Line Markers . .	81
4.23	First 10 MC Simulations of FX GARCH(1,1) Models + Mean (In Bold Red) for 2005	83
4.24	Last 10 MC Simulations of FX GARCH(1,1) Models + Mean (In Bold Red) for 2005	85
4.25	FX GARCH(1,1) Predicted Conditional Standard Deviation for 2005	87
4.26	FX GARCH(1,1) Volatility MC Simulated Confidence Interval for 2005	89
4.27	First MC Simulation of FX GARCH(1,1) Models Netted for 2005 for 5 Banks . .	98
4.28	First MC Simulation of FX GARCH(1,1) Models Netted for 2005 for 4 Banks . .	99
4.29	MC Simulation Distribution of 250 FX GARCH(1,1) Models Netted for 2005 for 5 Banks	101
4.30	MC Simulation Distribution of 250 FX GARCH(1,1) Models Netted for 2005 for 4 Banks	102
5.1	Schematic Example of Close-out Netting When the Exposures are Far Appart, and When they are Close Together	108
5.2	500 R250 Normalised 2D-Plot Pairs (x_n, x_{n+1}) Plot	114
A.1	Exposures Between Three Banks	124
A.2	Exposures Between Four Banks	127
A.3	Exposures Between Five Banks	128
B.1	Conceptual Diagram of the Density Function of a Typical Bank's Loss Distribution.	133
C.1	500 MATLAB (R250) Pseudo-Random and 500 Halton Quasi-Random Numbers in a (x_n, x_{n+1}) Plot or Marsaglia Plot	138
C.2	500 Halton Quasi-Random Numbers of Figure C.1 in a (x_n, x_{n+1}, x_{n+2}) Plot . . .	139

List of Tables

4.1	FX Rate Summary Statistics for 2004	55
4.2	FX Rate ARCH Test Summary Statistics for 2004	55
4.3	Distribution Specifications for USD/EUR	57
4.4	USD/AUD GARCH(1,1) Parameter Estimates for 2004	74
4.5	USD/CAD GARCH(1,1) Parameter Estimates for 2004	75
4.6	USD/EUR GARCH(1,1) Parameter Estimates for 2004	76
4.7	USD/GBP GARCH(1,1) Parameter Estimates for 2004	77
4.8	USD/JPY GARCH(1,1) Parameter Estimates for 2004	78
4.9	First 10 MC Simulations of FX GARCH(1,1) Models for 1st of 250 Simulation Runs for 2005 in Business Notation	90
4.10	First 10 MC Simulations of FX GARCH(1,1) Models for 1st of 250 Simulation Runs for 2005 in Netting Notation	90
4.11	Two Sample t-Test for 5 Banks Both With and Without Netting	103
4.12	Two Sample Wilcoxon Rank-Sum Test for 5 Banks Both With and Without Netting	103
C.1	The Advantages and Disadvantages of Monte Carlo and Quasi-Monte Carlo Sim- ulation (Algorithmics, 2000).	136

Chapter 1

Introduction

This thesis examines the underlying nature of netting on bank portfolios. Netting is the process in which two business entities that owe each other money, agree to offset their amounts and record just the difference. For example, if Bank A owes Bank B \$100,000 and Bank B owes Bank A \$20,000, one can effectively net the amounts and record that Bank A owes Bank B \$80,000 and Bank B owes Bank A \$0. The focus in this thesis is on examining specifically ‘close-out’ netting as opposed to the other two forms of netting; Payment/Settlement Netting and Novation Netting. Payment/Settlement Netting is the process of settling all deals between two counterparties on a net basis (Corrigan, et. al., 1999). Novation Netting is the process of replacing a set of obligations involving gross payments/deliveries with another set of obligations providing for net payment/delivery (Corrigan, et. al., 1999). In Close-out Netting, business entities enter a stringent legal agreement where they record their exposures (or amounts of money lent that are exposed to credit default risk) as if either business entity was bankrupt. This means that they are legally required to guarantee that at least the net amount will be available to the appropriate entity.

This thesis will also focus on banks as the main type of business entities. This is because banks were the first business entities to adopt close-out netting in a large scale, and so their practice and corresponding literature is the most extensive. As such we will examine netting within the framework of the Basel II Accord of international bank regulations (Basel Committee, 2001). The topic is financial in nature, and can be considered with either a predominantly qualitative perspective, by focusing on the legal and accounting aspects, or take a predominantly quantitative perspective, by focusing on the economics and financial econometrics aspects. This thesis will address the topic through a balanced approach.

The major reason why international banks become bankrupt is due to the volatility of interest and foreign exchange (FX) rates, and their impact on financial instruments. We will examine this phenomenon by simulating the stochastic volatility of FX rates for various arrangements of banks. We will simulate first for two banks, then three, and then generalise to multiple banks. By simulation, we mean Monte Carlo (MC) Simulation, in which suitably distributed random numbers are passed into the netting model to determine the behaviour of the model and how it effects the outcomes. Conversely, this approach also estimates the sensitivity of the netting model to sudden changes in the input numbers. By doing so, we can address the question of ‘How much does netting reduce the risk of sudden changes in FX rates’? The way in which the distributions of these numbers will be controlled and analysed will be through the widely accepted Generalised Auto Regressive Conditionally Heteroschedastic (GARCH) methodology. Although we will not delve too deep into GARCH, we will be able to leverage some of its features that will help assess the nature of close-out netting in a realistic financial framework.

The reason why close-out netting is being examined here is that there has been little previous research in this area, despite its benefits (Fermanian and Scaillet, 2003). Such benefits arise due to close-out netting minimising the amount of money that is exposed to another business entity. As a result, less regulatory capital for capital adequacy is required, which can be released for reinvestment activities. It will thus be very beneficial to research the underlying nature of close-out netting to help measure or estimate its benefits. At the heart of these issues, there is a central question which is stated separately below.

Research Question

The question that this thesis aims at answering can be expressed as “What is the Underlying Nature and Mechanics of Close-out Netting?”

The stochastic volatility of daily foreign exchange (FX) derivatives poses a number of risks for the international banking community. Settlement risk, liquidity risk and capital adequacy are just a few immediate concerns that arise from such volatility.

This thesis examines the impact of close-out netting on minimising the stochastic volatility of inter-bank FX derivatives. The problem with close-out netting is that although it is a simple formula of taking the differences between two banks at one point in time, it is the stochastic

and volatile nature of FX rates that makes measuring the full impact of netting difficult.

The objective of this thesis is to establish a realistic international banking framework or modelling environment in which close-out netting can be scientifically applied and examined. Five international daily FX rates will be used as sufficient approximations for five international banks. A generalised autoregressive conditionally heteroschedastic (GARCH) modelling approach is adopted as a robust and rich FX volatility paradigm. Then through Monte Carlo simulation of the resulting fitted GARCH models, we generate the distributions -with and without close-out netting. Through statistical techniques, we estimate the impact FX volatility due to close-out netting.

Having introduced what, how and why the netting research has been carried out, we can now define each of the core concepts separately.

1.1 Close-out Netting

Close-out netting is a risk minimisation technique that applies mainly to the lending and borrowing of large sums of money between banks (i.e. bank counterparties). Netting can also be utilised by large corporates and multinational financial institutions due to their large monetary reserves. The following defines close-out netting as per the Australian Netting Act of 1998.

Definition 1 (Close-out Netting)

For close-out netting to occur, the following requirements are to be satisfied (“Netting Act”, 1998);

- (a). Parties close-out (or formally terminate) each transaction existing between them before the scheduled maturity date by determining a **present value** to each transaction Mark-to-Market (MtM) ¹; and*
- (b). Each transaction is aggregated into a single net amount and transactions that are “In-The-Money” ² are netted against transactions that are “Out-Of-The-Money” ³ to arrive at an overall single sum.*

Whilst this definition of close-out netting is the Australian definition, it does not contain any

¹MtM will be defined in Section 1.2.

²In-The-Money will be defined in Section 1.2. under MtM

³Out-Of-The-Money will be defined in Section 1.2. under MtM

features that distinguish any Australian-specific regulations. In fact, although there is general legal consensus in various jurisdictions around the world that close-out netting is effective *pre-insolvency*, there is no such general consensus *post-insolvency*. This means that whilst netting is perfectly fine from an Accounting perspective, for it to be fully effective, the assets netted need to be securitised. Securitisation will be defined formally below but essentially it allows these assets to act as security in case of a credit default event. It is because of this ambiguity that before a close-out netting contract can be entered into, each counterparty's ⁴ legal representatives must ensure that all relevant documents are accurately submitted and are "in order" (APRA, 2000). This means that this Australian definition is generic enough to be applied to other countries legal context without loss of generality or cause of ambiguity.

Such a contractual arrangement has arisen due to many countries law courts wanting to eliminate the ambiguity that liquidators in a receivership can use to their clients advantage. Liquidators achieve this by choosing to claim on contracts that are in their clients favour, i.e. In-The-Money, but not on those not in their favour, i.e. Out-of-The-Money. This is referred to as "cherry picking" and has been phased out under close-out netting agreements (ISDA, 1999). Counterparties agreeing to close-out netting terms and conditions are able to mutually enjoy amongst other benefits, lower risks by knowing upfront each counterparty's total exposure to one another and consequently are also better able to manage their ongoing capital allocation and capital adequacy requirements (Basel Committee, 2001). However, there are disadvantages associated with close-out netting that the legal literature surprisingly doesn't capture. These issues are documented in **Chapter 2 - Literature Review**.

It is worthwhile clarifying that close-out netting involves finding the net difference between the two counterparties, but not in finding a counterparty for a matching amount. For example, if Bank A has written a \$10,000 contract with Bank B, then one needs to ask "How much money does Bank B owe Bank A?", or equivalently "What exposure does Bank B have with Bank A?", but not "Which Bank owes Bank A \$10,000 so that this can be netted off?".

Another point worth noting is that a bank is not so much concerned about situations where it cannot pay back a counterparty, but more if a counterparty cannot pay back the bank itself. Although such a stance for a bank to its counterparties may appear to increase the likelihood of credit defaults to their counterparties, close-out netting actually decreases this particular like-

⁴A counterparty is the business entity that one has an exposure to and is also known as an obligor or creditor.

likelihood. This is achieved by recording wholesale transactions with close-out netting factored in, resulting in the net amounts being reconciled. Because amounts lent and borrowed are combined in this manner, exposures are always positive (i.e. greater than \$0), and are recorded against the counterparty with the greatest exposure to the other.

It is possible that the simplicity behind close-out netting has lead researchers to believe that there is not much that can be progressed in close-out netting and hence the scarcity of research in this area. Another reason why there are very few quantitative studies on the impact of close-out netting is due to its relatively recent implementation, and that it is a strategy that is mainly used commercially. As mentioned above, this thesis will focus on close-out netting rather than the other forms of netting. This is because it is the most recently introduced form of netting and also because it refers to credit defaults and hence credit risk analysis.

1.2 Other Definitions

Definition 2 (Mark to Market [MtM])

Mark to Market (MtM) is a measure of the replacement cost of contracts in the market. It assesses the In- or Out-of-The-Money position of a contract and when the measure is;

- (a). Positive or In-The-Money, the counterparty has credit exposure against the bank.; and*
- (b). Negative or Out-of-The-Money, the bank has credit exposure to the counterparty.*

(Holton, 2001).

The methodology or approach that this thesis will take in modelling netting's impacts will be through simulation. In particular, MC Simulation will be adopted as opposed to other forms of simulation.

Definition 3 (Monte Carlo (MC) Simulation)

A method of generating values from a known distribution for the purposes of experimentation. This is typically accomplished by generating uniform random variables and using them in an inverse reliability equation to produce failure times that would conform to the desired input distribution Chinarel Knowledgebase, 2005).

Yet another definition of MC simulation is,

A technique used in computer simulations that uses sampling from a random number sequence to simulate characteristics or events or outcomes with multiple possible values. For example, this can be used to represent or model many individual patients in a population with ranges of values for certain health characteristics or outcomes. In some cases, the random components are added to the values of a known input variable for the purpose of determining the effects of fluctuations of this variable on the values of the output variable (NICHSR, 2005).

Finally, the other main concept that needs to be defined is stochastic processes, as they describe the nature of FX rates and their volatility.

Definition 4 (Stochastic Processes)

In the mathematics of probability, a stochastic process is a random function. In the most common applications, the domain over which the function is defined is a time interval (a stochastic process of this kind is called a “time series” in its applications) or a region of space (a stochastic process being called a random field). Familiar examples of time series include stock market and FX rate fluctuations, signals such as speech, audio and video; medical data such as a patient’s EKG, EEG, blood pressure or temperature; and random movement such as Brownian motion or random walks. Examples of random fields include static images, random topographies (landscapes), or composition variations of an inhomogeneous material (Wikipedia Knowledgebase, 2006).

We can now outline the remaining structure of the thesis.

1.3 Thesis Structure

The remaining chapters build on this introduction by providing progressive levels of detail whilst narrowing our focus on the impacts of close-out netting. The rest of this thesis is organised as follows.

Chapter 2 - Literature Review

This chapter reviews the literature behind close-out netting by examining the Institutional and Historical Framework of Netting, Netting Legislation, Credit Risk Models and finally Netting Research. It provides a context for the research, how this thesis relates to other research, and

highlights that there is little quantitative research on close-out netting.

Chapter 3 - Modelling Methodology

This chapter explains how the data was selected and captured so that it could be modelled for FX rate volatility. It then describes the Generalised Auto-Regressive Conditionally Heteroschedastic (GARCH) methodology to generate volatile credit exposures that are netted by the netting methodology. Finally, the Monte Carlo simulation methodology is discussed to obtain sufficient scenarios of the netted GARCH exposures. It also generalises these results through the use of more theoretical analyses.

Chapter 4 - Modelling Results

This chapter lists the results and analyses their characteristics in detail. These include FX rate time series data, GARCH models, MC GARCH simulations, applications of the netting model. These sections all undertake distributional analysis to validate that the results are not incorrect.

Chapter 5 - Conclusions

This chapter concludes the research by summarising the results, relating the results back to the original research question. The results are presented objectively to minimise missinterpretation. It also proposes other related aspects of research that are outside the scope of this thesis, yet still worthwhile addressing.

Chapter 2

Literature Review

The netting literature is important as it gives context and forms a foundation for the rest of the thesis. Finding literature on close-out netting is also difficult due to it being only relatively recently introduced in Australia, in 2000, and only slightly earlier in other countries. The fact that there is earlier literature on other forms of netting means that certain relevant articles and papers have already been written, but under alternate names. Sometimes close-out netting is labelled as “Netting Under Default”, and “Netting by Close-out”. Other times it will be translated into English as “Offsetting”, hence confusing the reader with the related yet inappropriate “Offset Accounting”. Much of the literature on close-out netting is couched in a legal context since fundamentally it is a legal concept that is defined and enforced by the law. This chapter has been divided into “Institutional and Historical Framework of Netting”, “Netting Legislation”, “Credit Risk Models” and “Netting Research”.

2.1 Institutional and Historical Framework of Netting

We begin by detailing the volatile and risky nature of financial institutions so that later we can see how and why netting is a process that can minimise the associated risks. These risks mainly arise due to sudden fluctuations in FX rates and also in domestic and international interest rates. Each day enormous amounts of money are transferred between financial institutions around the world as settlement for FX transactions and interest rate derivatives/instruments. Owing to time zones and technological limitations, parties to settlement transactions usually assume full and unsecured risk with regard to counterparty exposure to settlement risk ¹. A bank’s risk

¹Settlement risk is defined below.

exposure for each transaction often lasts for as long as 48 hours, sometimes entailing credit risk against counterparties which exceeds the bank's equity or capital (Allsopp Report, 1996). Over the past 25 years, liberalisation and internationalisation of capital markets, combined with advances in FX trading technology, have led to considerably stronger growth in FX trading than implied by the growth in international trade in goods. According to an international survey carried out by the Bank for International Settlements (BIS) in 1998, daily FX trading had reached an estimated USD \$1,500 billion (BIS, 1998). FX transactions form the largest sector of international finance, and so this thesis will restrict its focus to FX instruments and rates.

FX transactions involve the payment of the sold currency to a counterparty in return for receipt of the bought currency from that counterparty. For example, when buying USD against JPY, the trade is settled with a payment of JPY against receipt of USD. However, FX derivatives are more complex than just the exchange of currency since they also involve margin calls, foreign and domestic interest rates. These are some of the reasons why sometimes banks or their counterparties default on their settlement amounts, or even worse, through to being declared bankrupt. FX transactions also give rise to various forms of risk and these types of risk include Settlement, Market, Credit, Systemic and Operational risk. These are discussed below.

Settlement risk is also known as Liquidity, Payment or Herstatt risk, and can be narrowly defined as “the risk of loss when a bank in a FX transaction pays the currency it sold but does not receive the currency it bought” (BIS, 2000 p1). A loss would constitute a bank or counterparty delaying a payment or settlement amount. The consequences of this range from incurring losses such as fines, through to a downgrade in credit risk rating (making it more expensive to do business with that bank). This type of risk will be the main focus of this thesis, as it is the most common type of risk in FX transactions. If this were to occur over an extended period of time, it would then be classified as a default and hence pertain to credit risk.

Market risk is defined as “the risk of losses in On- and Off-balance sheet positions arising from movements in market prices, including interest rates, FX rates and equity values” (BIS, 1996 p45). Market risk mainly involves relatively high probabilities that one will lose relatively low amounts of money. In the long run, investment returns that are subject to predominantly market risk exhibit a symmetrical normal distribution (Morris, 2000). From a more detailed perspective, we observe that these financial returns exhibit leptokurtosis, i.e. a much higher peak with “fat tails” than the normal distribution (Premaratne and Bera, 2005). A loss would

constitute a reduction in the instrument's price based on its demand.

Credit risk or credit default risk, is defined as “the exposure to uncertainty in a counterparty's ability to meet its obligations” (Marphatia and Tiwari, 2000 p7). Credit risk mainly involves relatively low probabilities that one will lose relatively high amounts of money. In this sense, credit risk is almost the exact opposite to its market risk counterpart. In the long run, and from a high-level overview perspective, investments that are subject to predominantly credit risk exhibit a right-skewed lognormal distribution (Morris, 2000). These distributions typically display high kurtosis requiring more sophisticated techniques. These shall be discussed in the next chapter via the GARCH methodology. A loss would constitute a counterparty not being able to repay their debt.

System or **systemic risk** is defined as “the risk that the inability of one participant in a payment system, or in a financial market, to meet its obligations when due will cause other participants to fail to meet their obligations when due” (Marphatia and Tiwari, 2000 p7). A loss by one bank could cause a cascading effect across multiple banks that did not necessarily have any transactions with that original bank.

Operational risk is defined as “the risk of direct or indirect loss resulting from inadequate or failed internal processes, people and systems or from external events” (BIS, 2001 p2). As an example, a lack of adequate internal controls in a bank could lead to human errors not being validated, through to fraudulent activities not being detected.

Since international banks typically have a low likelihood of having credit risk and indeed bankruptcy, credit risk will not be modelled in this thesis, but we will survey the common types of credit risk models. This was a further reason to focus on FX settlement risk. Another reason was that changes in market risk often have flow on effects with settlement risk, so to avoid multivariate or convoluted effects clouding the results, a clearer and simpler focus was adopted. A more formal definition of FX settlement risk than the definition above was adopted by the Committee on Payment and Settlement Systems (CPSS) and is set out as follows.

“A bank's actual exposure -the amount at risk -when settling a FX trade equals the full amount of the currency purchased and lasts from the time a payment instruction for the currency sold can no longer be cancelled unilaterally

until the time the currency purchased is received with finality” (Allsopp Report, 1996 p8).

As mentioned previously, FX settlement risk can be minimised by netting. It is thus important to discuss the legal framework that supports netting activities.

2.2 Netting Legislation

Netting has had a dramatic impact on international banking law and indeed Australian law. The list below details how for example the Australian “Netting Act” is to expressly override the existing Australian law (Stumbles and Kerr, 1998). Although it is not the scope of this thesis to delve too deep into the legal aspects of netting, the list below shows that many legal documents have had to be modified to accommodate netting. A list of these Acts include; Section 11F and 16 of the Banking Act, 1959; Section 187 of the Life Insurance Act, 1995; Section 86 of the Reserve Bank Act, 1959; Sections 437D, 468 and the preference provisions in Division 2 of Part 5.7B of the Corporations Law; Sections 120, 121 and 122 of the Bankruptcy Act, 1966; Section 142 of the Superannuation Industry (Supervision) Act, 1993. Despite netting’s many benefits and changes to legal documents, there are certain disadvantages associated with netting that the legal literature does not address in detail. These issues are introduced later in this chapter under the section Netting Research.

Since netting significantly reduces the various forms of risk by minimising the amount of money that is exposed, banks that utilise netting thus have less restrictive capital adequacy requirements, as provisioned by the international Basel II Accord (BIS, 2001). APRA and the federal Reserve Bank of Australia are then responsible amongst other tasks for setting the guidelines to calculate minimum regulatory capital reserves. Additional references on Australian specific capital adequacy and capital allocation that have been affected by the “Netting Act” include the following two key APRA documents.

The first of these documents on capital regulation is the Prudential Statement C1: Capital Adequacy of Banks (APRA, 1999). The relevance of this is that all banks are required to hold a certain minimum level of capital reserve in case they declare bankruptcy or they begin defaulting on their credit obligations to their counterparties. The size of this capital reserve is mainly dependant upon both the amount of the default, and the number of defaults. This capital reserve is also dependant on the degree of financial backing of the bank, and on its credit rating.

The second of these documents on capital regulation is the Prudential Statement E1: Supervision of Bank's Large Credit Exposure (APRA, 1989). The relevance of this is due to the way in which banks are more heavily regulated than multinational corporates and other financial institutions. This document details how APRA is required to work in conjunction with the federal Reserve Bank of Australia (RBA) to ensure that Australian banks meet their national and international regulations. This has impacted banks by forcing them to have greater transparency and disclosure in how they report large exposures as Current Liabilities on their Balance Sheets.

"The Basel II Accord" (BIS, 2001) is directed by the Basel Committee, which is the international equivalent of APRA for banking regulation and supervision. The "Report on Netting Schemes" (Angell, 1989) is a primer on netting. It presents an analysis of the credit and liquidity risks experienced by participants in bilateral and multilateral netting arrangements for both interbank payment orders and forward-value contracts such as FX trades. The Report also looks at the effects of netting on the integrity of interbank settlement arrangements, the conduct of monetary policy, the complications posed for the allocation of supervisory responsibilities, and the effective oversight of cross-border netting schemes.

One year later, BIS issued "Minimum standards for the design and operation of netting schemes" (Lamfalussy, 1990). It introduced six standards for netting systems to adhere to, and these are:

1. Netting schemes should have a well-founded legal basis under all relevant jurisdictions.
2. Netting scheme participants should have a clear understanding of the impact of the particular scheme on each of the financial risks affected by the netting process.
3. Multilateral netting systems should have clearly defined procedures for the management of credit risks and liquidity risks that specify the respective responsibilities of the netting provider and the participants.
4. Multilateral netting systems should, at a minimum, be capable of ensuring the timely completion of daily settlements in the event of an inability to settle by the participant with the largest single net-debit position.
5. Multilateral netting systems should have objective and publicly-disclosed criteria for admission which permit fair and open access.
6. All netting schemes should ensure the operational reliability of technical systems and the availability of back-up facilities capable of completing daily processing requirements.

Further to these standards is the “Report of the Committee on Interbank Netting Schemes of the Central Banks of the Group of Ten Countries” (BIS, 1990). The document analyses the impact of netting on credit, liquidity and systemic risk and sets out the principles for central bank oversight of cross-border and multi-currency netting and settlement schemes. Prior to netting, exposure for each N assets for $i \in \{1, \dots, N\}$ in a portfolio (A_{Net}) was approximated by taking the weighted average of the underlying gross amounts (A_{Gross}) _{i} giving,

$$A_{Net} = \frac{1}{N} \sum_{i=1}^N (A_{Gross})_i. \quad (2.1)$$

To account for each M netted assets for $j \in \{1, \dots, M\}$, an add-on was included in Equation 2.1 giving,

$$A_{Net} = \frac{w_1}{N} \sum_{i=1}^N (A_{Gross})_i + \frac{w_2 \times NGR}{M} \sum_{j=1}^M (A_{Gross})_j, \quad (2.2)$$

where w_1 and w_2 are weighting factors, and NGR is the current netted market or replacement value divided by current grossed market or replacement value (BIS, 1995). Initially the weights were set to $w_1 = 0.5$ and $w_2 = 0.5$ as the add-on component was a significant improvement to the calculation. Later, the netting effect was found to have a more prominent impact, and so the weights were set to $w_1 = 0.4$ and $w_2 = 0.6$.

An attempt to extend bilateral netting by taking into account the effects of multilateral netting was then documented in “Interpretation of the Capital Accord for Multilateral Netting of Forward Value Foreign Exchange Transactions” (BIS, 1996). It suggested that a participating counterparty’s capital requirement for current credit exposure was the sum of the participant’s pro-rata share of the clearing house exposure that it would be required to absorb from a default by every other participant, individually, in the clearing system.

The “Consultative Paper on On-Balance Sheet Netting” (BIS, 1998) extended the scope of netting beyond derivatives to On-Balance Sheet items, such as deposits and loans. It proposed to accept novation netting as a means of reducing gross exposures to a single net amount, provided the following four conditions are met; First, the reporting bank has a well-founded legal basis for concluding the netting or offsetting and the agreement is enforceable in each relevant jurisdiction. Second, the maturity of the deposit is at least as long as the corresponding loan.

Third, the positions are denominated in the same currency. Fourth, the reporting bank monitors and controls the relevant exposures on a net basis.

This prompted the Basel Committees' and the IOSCO Technical Committees' concerns about supervisors receiving sufficient information on a bank's derivative activities, causing them to issue "Framework for Supervisory Information about Derivatives Activities of Banks and Securities Firms" (1995). The first part of this document examines how banking regulators require information on the firm's Value at Risk (VaR) models as well as summary VaR numbers (high/low/average). The second part consists of a common minimum framework which the committees regard as the baseline of disclosure requirements for large and internationally active banks. The Committee also recommended the disclosure of Summary VaR and/or Earnings at Risk (EaR) information and scenario analysis of the impact of rate shocks for non-traded portfolios. Firms are also expected to give information about the effect of off-balance sheet positions on their earnings.

2.3 Credit Risk Models

BIS rewards banks for modelling credit risk and for openly reporting on the performance of these models. There are four main types of credit risk models used in the banking industry. Three papers that review these models are Crouchy, Galai and Mark (2000), Phelan and Alexander (1999) and Basel Committee (1991). These will not be discussed in great detail since international banks are more subject to settlement risk than credit risk, and so rarely go into liquidation. Nevertheless, since we will be measuring exposure before netting and then after netting, the use of the most popular industry accepted credit risk models will facilitate a structured approach and link together many of the topics to be discussed. The four main model categories are listed below.

The KMV PortfolioManager Model by KMV Corporation (Vasicek, 1987) essentially uses a database of thousands of banks' historical financial data forming multiple time series. By using measures such as Loss Given Default (LGD) and Expected Default Frequency (EDF), PortfolioManager applies covariance matrices to estimate a bank's Joint Default Frequency (JDF). Explicitly calculating this relationship requires the calculation of a double integral (over all pos-

sible asset values) for each pair of banks in the portfolio. Even for the average sized portfolio, this approach is cost prohibitive from a computational perspective, and so MC simulation is often used to estimate these JDFs. This adds to the literature that recommends the adoption of the MC method in modelling credit and settlement risk.

The CreditMetrics Model by J.P. Morgan (1997) is the credit risk counterpart to the RiskMetrics model, where RiskMetrics is predominantly a market risk model, also from J. P. Morgan Ltd. Although RiskMetrics is based on Merton's (1974) work on the relationship between a counterparty's capital structure and insolvency risk, CreditMetrics only makes use of the basic idea from this approach. CreditMetrics achieve this by simulating correlated standard-normally distributed random numbers. This supports the use of simulation methods (including MC) and supports the netting research of Fermanian and Scaillet (2003) in which greater correlations within a portfolio's asset group leads to greater risks, both with and without netting.

The CreditRisk+ Model by Credit Suisse First Boston (1997) is also known as the actuarial risk model. Only credit risk from defaults is considered, allowing for a more closed-form analytical solution in the form of stochastic differential equations (SDEs) and the use of Poisson distributions. Since credit default rates are reported in the finance industry predominantly as discrete values, the continuous SDE model outputs are discretized.

Finally, the CreditPortfolioView Model by McKinsey and Company (Wilson, 1997) is also known as the econometric risk model. Its concept can be seen to lie in between that of the CreditRisk+ and the CreditMetrics models. Default rates though, are calculated in a more sophisticated manner by using complete time series of various macroeconomic variables such as FX rates to simulate future events. It is based on the causal observation that default probabilities and migration probabilities are directly impacted when the economy worsens, which results in down grades as well as increased credit defaults.

These credit risk models have MC simulation as a common similarity in the way in which their models are numerically approximated.

2.4 Netting Research

It is rare to find quantitative research papers that discuss netting generally, let alone any of the types of netting. As mentioned in a previous section, most of the sparse literature on netting is qualitative and expressed in a legal context. The following research papers discuss the quantitative aspects of netting and their mathematical approach provides a deeper insight into the mechanics of netting itself.

Gizycki and Gray (1994) discuss netting impacts on primarily swap and Forward Rate Agreement (FRA) portfolios of Australian banks. The authors mention that there are essentially two simple ways to look at the behaviour of netting on potential exposure within portfolios. The first of these is the worst case scenario basis, and the second is the statistical analysis of portfolio value changes over time. The worst case scenario basis involves simulating exposure values that have high correlation so that the netting impact can be observed at its maximum. The statistical analysis basis involves looking at the potential exposure by considering changes in net replacement costs, i.e. the costs required to replace the portfolio in case of a default. Whilst netting may, therefore, reduce the level of current credit risk of a portfolio, it says little about the potential exposure, as it is determined by changes in the net replacement cost. Gizycki and Gray conclude by reinforcing the importance of Add-Ons by referring to the work by Estrella and Hendricks (1992), who tested how well Add-Ons that were based on notional principals and net replacement values, covered potential exposure as the proportion of the portfolio having netting increases.

Fermanian and Scaillet (2003) is by far the most complete work available on modelling the impact of netting. The authors establish a fundamental framework for their research which helps other researchers to see the link between financial econometrics and netting, and how mathematics can be used to obtain new results on netting. The authors also agree with the claim of this thesis, that papers on netting are not quantitative enough. Not only is netting discussed, but also close-out netting in particular. VaR and Expected Shortfall (ES) are analysed both with and without netting to determine the changes due to netting.

Through the use of advanced analysis theory, Fermanian and Scaillet apply Lebesgue Integration and Non-Parametric Kernel Regression Estimators to estimate VaR and ES . In this paper, VaR and ES are explained in terms of underlying portfolio asset components and also

in terms of the corresponding portfolio loss functions. This allows for many netting concepts to be formulated and tested. Fermanian and Scaillet point out that taking partial derivatives of VaR and ES under netting would better allow risk managers to isolate high risk asset groups because the portfolio is consequently decomposed into its asset group constituents. The authors detail that for each of the j generalised partial derivatives of VaR and ES with respect to some asset ϵ , i.e. $\partial_{\epsilon}VaR_j$ and $\partial_{\epsilon}ES_j$ then $j \in \mathbb{N}$ requires the evaluation of underlying truncated distributions. Although these distributions tend to be Gaussian as the time horizon approaches infinity, $\partial_{\epsilon}VaR_j$ and $\partial_{\epsilon}ES_j$ themselves do not admit explicit forms (even for low values of j). As a result, they need to be computed numerically, hence providing further reasoning for the use of MC Simulation techniques. Finally, the authors plot the relationship between VaR and VaR sensitivity on correlation, both with and without netting. They conclude that netting reduces or smooths out the sensitivity effects on VaR as correlation amongst contracts increases.

Emmons (1995) is the second most complete quantitative work on netting, after that of Fermanian and Scaillet (2003). Emmons begins by stating that whilst netting is essentially a beneficial and worthwhile risk management strategy, it is not quantitatively documented in the scarcely available literature that it can increase the default risk of the bank creditors that have not entered into the netting agreement(s). Emmons points out how the US Federal Reserve Bank is aware of these risk shifts due to netting and so has implemented numerous “safeguards” such as debit caps, stringent collateral requirements, intraday monitoring and overdraft pricing. Emmons suggests that future research on netting impacts should analyse the trade off between netting and no netting. A formula needs to be found to determine optimal levels of netting for each individual bank after having taken an interbank analysis. Emmons mathematically shows how bilateral netting reduces credit default risk less than multilateral netting, yet more than gross netting. This is achieved by the application of the Law of Large Numbers, and basic limit convergence theorems. Emmons then extends the research by stating that in the limit, i.e. as the number of banks included in multilateral netting agreements increases without bound, the proportion of credit default risk borne by net interbank balances approaches zero. “In practice, a large multinational netting agreement may approach this limit with as few as one hundred members” (Emmons, 1995, p19). This convergence proposition was not proved by Emmons and so this thesis supplants Emmons’ research by investigated this claim as part of its MC simulation experiments and more generalised mathematical formulation of interbank netting schemes.

Wahrenburg (1997) outlines how aggregating portfolio asset group exposures can have counter-

intuitive effects on the overall portfolio exposure. Examples cited include unintentional transfers of risk between banks and some exposures not having large netted reductions. This is because of netting's subtractive ² properties can cause sudden reductions in a portfolio's overall credit exposure, making it more difficult to forecast trends. Discontinuities in the exposure curves due to netting results in lower predictive ability using conventional measures, increased variance and raises the need to reformulate VaR to account for these anomalies. At one extreme, the settlement agent may take no settlement risk. This, in effect, allocates the settlement risks of settlement to the participants in the system. At the other extreme a settlement agent could bear all of the settlement risks of settlement. For example, the "agent" could disburse settlement payments to net creditors before receiving amounts due from net debtors in a clearing, thus temporarily financing a settlement. This allocates all settlement risk to the "agent". However, this paper only looked at a specific example by basing its conclusions on two simplified swap contracts.

Levy and Clarke (2000) state that it is intuitively clear that netting reduces the risks of derivative transactions. Unfortunately, the computational modelling of netting impacts upon credit risk is much more challenging to establish and is not discussed at great detail. The paper shows how MC simulation can overcome many subtleties of the exposure calculations, thus avoiding an intractable calculation. Levy and Clarke claim that this approach represents an ideal trade-off between accuracy and feasibility for large derivative portfolios. They also mention that there is still much scope for research into the modelling of netting, hence supporting another claim of this thesis, that there are existing opportunities for new netting research.

Bech, Madsen and Natrop (2002) summarise how Chakravorti (2000) generalised the various forms of risk such as settlement and credit risk into a more general 'systemic' risk. They follow the framework of De Bandt and Hartmann (2000) who define a systemic event as an event where a shock to either a set of financial institutions or markets lead to considerable adverse effects on other financial institutions or markets. Based on this all encompassing definition of a systemic event, De Bandt and Hartmann defined systemic risk as the probability of such an event. Under such a generalised framework, the authors are able to quantify contagion and the domino effects within a netting environment. Although their matrix notation is elegant, it is only useful in describing the importance of netting, but not in measuring it. It is claimed that the severity

²The word subtractive is used here because netting is a process of subtracting the smaller of the two counterparty exposures from the larger.

of the systemic event depends on the amount of liquidity reserved, which in turn determines a threshold for netting impacts. The authors develop a trade-off between risk and liquidity amounts, but do not divulge their specific details. All that is left for the reader is to piece together that through an iterative approach of taking successive interpolations, the trade-off function can be approximated. As is common with most research on netting, the authors turn to simulation to estimate the severity of these risks under a netting environment. They conclude that performing sensitivity analysis on the simulated time series shows that the result of low systemic risk is robust to large changes in the liquidity available to the banking participants under netting. Although the paper focuses on the Danish financial system, it does support that netting reduces the impact of banking risks.

This chapter has reviewed the institutional banking and historical framework surrounding netting and the credit risk models that are used within this banking industry setting. So far, the scarcity of quantitative research on close-out netting suggests that a controlled environment or modelling framework should be established and enhanced to test various hypotheses. Having revised the available literature relating to netting, both quantitative and qualitative, sufficient context has been provided to commence the modelling methodology in the next chapter.

Chapter 3

Modelling Methodology

This chapter will build on the literature review of Chapter 2 by addressing the modelling methodology. Since netting minimises the impact of the volatility of asset returns, we will measure or estimate the nature in which this occurs. By volatility, we mean the stochastic volatility of FX rates amongst international banks, and not the volatility of bank's domestic interest rates. If we were to, for example, look at the volatility of Australian bank's domestic interest rates, then they would all have similar interest rates. This is because bank's interest rates are based on the Cash Rate which in Australia is determined by the federal RBA. Such an approach of just analysing one countries banks would not maximise the presence of volatility and would hence minimise the impact of netting.

A natural approach to the modelling would be to try and replicate various international bank's derivatives portfolios and then net them off. However, such an approach would complicate matters unnecessarily because one would need to determine each bank's asset group composition in terms of the asset types and their concentrations. Consider if there was then a sudden FX rate volatility jump in the portfolio performance's time series. It would then be much more difficult to isolate whether such a change was due to the underlying volatility of the FX rate, or the sensitivity of a particular asset class to the FX rate change. It is thus a much "cleaner" approach to model the FX volatility itself, given that the objective of this thesis is to examine netting's ability to minimise risk rather than the details of the risk itself.

The most popular and accepted method of modelling FX volatility is the GARCH methodology (Hansen and Lunde, 2001). We can then pass modelled GARCH exposures through the netting model, which will be discussed in detail below. MC simulation can ensure that we have

taken sufficient GARCH simulations to ensure that our netted time series are representative of the possible exposures. The sections of this chapter will be Data Preparation, Stochastic Volatility of FX Rates Methodology, GARCH Methodology, Netting Methodology, and MC Simulation Methodology.

3.1 Data Preparation

This thesis has chosen to analyse the impact of FX volatility for five major currencies; Australian dollar (AUD), Canadian dollar (CAD), Euro (EUR), Great British pound (GBP), and Japanese yen (JPY). These currencies were chosen as they are frequently traded against the Australian dollar, whilst maintaining the US dollar (USD) as the standard. Although there are other frequently traded currencies such as the Chinese yuan (CNY), we restricted these to just five. This is because the benefit of including additional currencies/banks is out-weighed by the increase in cost of the calculations (Emmons, 1995). These calculations grow exponentially with every additional linear increase in number of currencies/banks due to the multivariate nature of netting. Any fewer than five FX rates would not adequately examine the multivariate ¹ impact of netting amongst these currencies.

The time interval for the data was chosen to be daily. This was because a smaller interval would not be too relevant as netting is calculated on a “close of business” basis (i.e. daily) (Lamfalussy, 1990). Choosing an interval smaller than daily would be difficult to obtain the time series data. It would also make it more difficult to find other sources to cross-validate the data, and would involve processing much larger volumes of data over the same time period. Furthermore, this would not be appropriate as we are trying to predict what may happen in one year’s time, not what will happen in a few week’s time. Such an approach would also involve too much noise or unstructured volatility. Likewise, choosing an interval larger than daily would be like aggregating daily FX rates into weekly, fortnightly or monthly totals, thereby reducing the volatility we are trying to model. We also do not want to predict what may happen in 5-10 years time either, hence daily data was chosen.

The time period for the data was two years, spanning 01/01/2004 to 31/12/2005. Two years was chosen so that the first year could be used for the model build or development, and the second year could be used for model validation or testing the predictions, as shown in Figure

¹This will become more apparent after the Netting Methodology section below.

3.1.

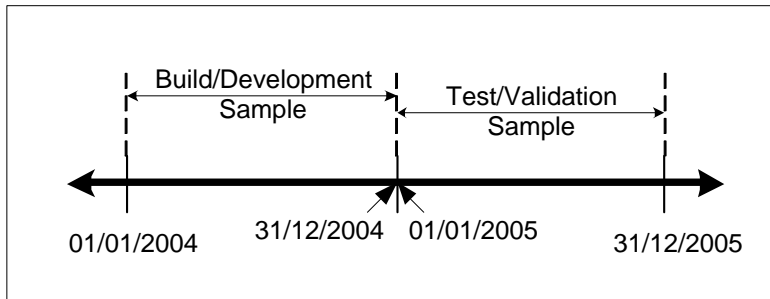


Figure 3.1: Use of Two Years of Daily FX Rate Data

Apart from the above justification for using two years of data, the reason why the same approach was not applied for say four or ten years worth, was that predicting daily data for one year out-of-sample would constitute 365 data points, which is adequate enough without being too excessive. Furthermore, as we will be using five FX rates, this would generate $365 \times 5 = 1,825$ data points. Since we will be repeating these projections over 250 MC simulations, this would then generate $1,825 \times 250 = 456,250$ data points. If we were then to predict two years out-of-sample, then this count would double to 912,500 data points, and so on. The reason why 2004 and 2005 data was chosen is due to these years being the most recent sets having the full yearly cycle of data available.

The underlying distributions were chosen to be the normal or Gaussian as most FX rate returns and their daily differences (i.e. $x_t - x_{t-1}$) can also be adequately approximated by this distribution (Andersen, Bollerslev *et. al.*, 1999). The Gaussian is also frequently used in GARCH specifications. Later we will find that for one of the FX rates, a better choice was found than the Gaussian.

3.2 Stochastic Volatility of FX Rates Methodology

In finance, the FX rate between two currencies specifies how much one currency is worth in terms of the other. The Currency Market or FX Market is the largest market in the world. By some estimates, about \$2 trillion USD worth of currency changes hands every day (BIS, 1998).

The general class of continuous-time FX models of currency return volatility can be written as a stochastic differential equation of the type,

$$dX_t = \mu[X_t; \Theta]dt + \sigma[X_t; \Theta]dW_t, \quad (3.1)$$

where W_t is a Brownian motion, $\mu[X_t; \Theta]$ is the risk neutral drift function, and $\sigma[X_t; \Theta]$ is the diffusion function. $\mu[X_t; \Theta]$ and $\sigma[X_t; \Theta]$ are assumed to be functions of the state variable X and some unknown parameter vector $\Theta \in \mathbb{R}^K$ (Ullrich, 2005).

In the study of FX returns, it has been a known fact that the return itself can not be predicted (Yang, 2003). It is the forecasting of the returns' volatility that is of special interest. By returns, we mean daily difference data as per above. As a time series with zero conditional mean, the FX differences are conveniently modelled as a process $\{Y_t\}_{t=0}^\infty$ of the form,

$$Y_t = \sigma_t^2 \epsilon_t, \epsilon_t \in \mathbb{N}, \quad (3.2)$$

where the $\{\epsilon_t\}_{t=0}^\infty$'s are *iid* random variables independent of Y_0 and satisfying $E(\epsilon_t) = 0$, $E(\epsilon_t^2) = 1$, $E(\epsilon_t^4) = m_4 < +\infty$, while $\{\sigma_t^2\}_{t=0}^\infty$ denotes the conditional volatility series $\sigma_t^2 = \text{var}(Y_t | Y_{t-1}, Y_{t-2}, \dots)$.

At this stage, the reader may be anticipating a subsection on short term interest rates, long term (forward) interest rates, term-structure of interest rates, yield curves and other related areas. Although change in interest rates is important, this thesis is concerned with how netting minimises the effects of FX rate volatilities, and not so much on what are the drivers behind interest rate and FX volatility.

Empirical evidence has led to the understanding that σ_t^2 depends on infinitely many past returns $Y_{t-j}, j \in \mathbb{N}$, with diminishing weights (Yang, 2005). The GARCH(p,q) model of Bollerslev (1986), for example, allows the volatility function to depend on all past observations, with geometrically decaying rate, the most commonly used is the GARCH(1,1) model.

3.3 GARCH Methodology

Before moving onto GARCH models, we begin by stating that the more simpler version that takes account of financial time series characteristics is the Autoregressive Conditional Heteroscedasticity (ARCH) process (Engle, 1982). ARCH models are generally couched in terms of asset returns or differences. The ARCH model is a technique used in finance to model asset price volatility over time.

It is observed in many time series data on asset prices that there are periods when variance is high and periods where variance is low. The ARCH econometric model for this is that the variance of the series itself is an AR (autoregressive) time series, often a linear one. Formally, per Bollerslev *et. al.* (1992) and Engle (1982), an ARCH model is a discrete time stochastic process e_t of the form $e_t = z_t s_t$ (i.e. as per Equation 3.2), where the z_t 's are *iid* over time, $E(z_t) = 0$, $var(z_t) = 1$, and $s_t > 0$ and time-varying. Usually s_t is further modelled to be an autoregressive process. According to Andersen and Bollerslev (1995, 1996, 1997), ARCH models are usually estimated by maximum likelihood techniques. They almost always give a leptokurtic distribution of asset differences even if one assumes that each period's returns are normal, because the variance is not the same each period. More generally speaking, the ARCH(q) model expresses the conditional variance as a linear function of the past q innovations,

$$\sigma_t^2 = a_0 + \sum_{i=1}^q a_i \epsilon_{t-i}^2. \quad (3.3)$$

For the conditional variance to be positive, the parameters must satisfy $a_0 > 0$ and $a_i \geq 0$ for $i = 1, \dots, q$. Empirical evidence has shown that a high q must be selected in order to estimate the conditional variance properly. To circumvent this problem, Bollerslev (1986) proposed the generalized ARCH, or GARCH(p, q), model,

$$\sigma_t^2 = a_0 + \sum_{i=1}^q a_i \epsilon_{t-i}^2 + \sum_{j=1}^p b_j \sigma_{t-j}^2, \quad (3.4)$$

where $a_0 > 0$ and $a_i \geq 0$ for $i \in \{1, \dots, q\}$, and $b_j \geq 0$ for $j \in \{1, \dots, p\}$. If $\left(\sum_{i=1}^q a_i + \sum_{j=1}^p b_j\right) < 1$, then the process ϵ_t is covariance stationary and its unconditional variance is equal to,

$$\sigma^2 = \frac{a_0}{1 - \sum_{i=1}^q a_i - \sum_{j=1}^p b_j}. \quad (3.5)$$

A special case of the GARCH family is the Exponentially Weighted Moving Average (EWMA) alternative, used by the company RiskMetrics when they introduced the Analytic VaR methodology. The RiskMetrics model for volatility forecasting imposes the restrictions that $a_0 = 0$ and $\left(\sum_{i=1}^q a_i + \sum_{j=1}^p b_j\right) = 1$. In addition, the parameter $\sum_{j=1}^p b_j$ is not estimated, but imposed to be equal to 0.94 for daily data and go back 75 data points in their estimation horizon (Neely and Weller, 2001). The GARCH(p, q) model successfully captures several characteristics of financial time series, such as thick tailed returns and volatility clustering, as noted by Mandelbrot (1963 p394) "large changes tend to be followed by large changes of either sign, and small changes tend

to be followed by small changes”.

Results in Figlewski (1997) show that GARCH models fitted to daily data are quite useful in forecasting the volatility in stock markets both for short and long horizons, although they are much less useful in making out-of-sample forecasts in other markets beyond short horizons. Berkowitz and O’Brien (2002) report that the poor performance of banks’ trading portfolios during the 1998-2000 period was mainly due to FX rate positions. Vlaar (2000) examines the out-of-sample forecast accuracy of several univariate volatility models (equally and exponentially weighted sample averages as well as GARCH models) for portfolios of Dutch bonds with different maturities. In addition, he examines the impact of alternative distributional assumptions. He concludes that VaR measures can be most simply and accurately specified using GARCH with a normal distribution and MC simulation.

Bond returns implied by interbank money market rates (LIBOR) can be used instead of bond returns. This is done in other studies such as Duffie and Singleton (1999) and presents several advantages. First, the interbank money market is more liquid than the Treasury bill market in most countries. Second, most FX rate derivatives are priced using interbank FX rates. Third, credit risk minimally affects these rates as they are subject to special contractual netting features and correspond to trades among high-grade banks.

On the other hand, the GARCH structure presents some drawbacks on implementation, since it (a) requires large numbers of observations to produce reliable estimates, and (b) may be unstable out of sample. It also imposes important limitations on the theoretical approach since variance depends on the magnitude and not on the sign of ϵ_t , which is somewhat at odds with the empirical behaviour of stock market prices where a leverage effect may be present.

There are a number of variations to the standard GARCH(p,q) model, including Nelson’s (1991) Exponential GARCH, or EGARCH(p,q) model and the Threshold GARCH, or TGARCH(p,q) model. This thesis will focus on the general GARCH(1,1) model as this has the most superior predictive ability at modelling mainstream FX rate effects (Hansen and Lunde, 2001). Hansen and Lunde make out-of-sample comparisons of the GARCH(1,1) model against 330 different volatility models using daily FX rate data (DEM and IBM stock prices). Although some models were better at forecasting one stock better than the other, no model provided a significantly better forecast than the GARCH(1,1) model. This result is established by the tests for superior

predictive ability of White (2000) and Hansen (2001).

3.4 Netting Methodology

This section details the mathematical framework to describe the netting interaction between banks. We begin by formulating the generalised netting model and then later provide an example of the model for two banks.

3.4.1 Generalised Netting Model

Let $\hat{e}_{i,j}(t)$ denote the exposure amount \hat{e} to be paid at time t from Bank i to Bank j as reported by Bank i . Within a netting scheme of n banks, the exposure matrix E is defined as,

$$E = \begin{bmatrix} 0 & \hat{e}_{1,2}(t) & \cdots & \cdots & \hat{e}_{1,n}(t) \\ \hat{e}_{2,1}(t) & 0 & \ddots & & \vdots \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & & \ddots & 0 & \hat{e}_{n-1,n}(t) \\ \hat{e}_{n,1}(t) & \cdots & \cdots & \hat{e}_{n,n-1}(t) & 0 \end{bmatrix}, \quad (3.6)$$

and captures all the gross exposure amounts for all the n banks participating within a netting scheme. Such an exposure matrix formulation was first documented by Emmons (1995).

We now define a Netting operator $N_1(*)$ and apply this to the netting scheme E such that,

$$\begin{aligned} N_1(E) &= N_1 \left(\begin{bmatrix} 0 & \hat{e}_{1,2}(t) & \cdots & \cdots & \hat{e}_{1,n}(t) \\ \hat{e}_{2,1}(t) & 0 & \ddots & & \vdots \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & & \ddots & 0 & \hat{e}_{n-1,n}(t) \\ \hat{e}_{n,1}(t) & \cdots & \cdots & \hat{e}_{n,n-1}(t) & 0 \end{bmatrix} \right) \\ &= \begin{bmatrix} 0 & \hat{e}_{1,2}(t) - \hat{e}_{2,1}(t) & \cdots & \cdots & \hat{e}_{1,n}(t) - \hat{e}_{n,1}(t) \\ \hat{e}_{2,1}(t) - \hat{e}_{1,2}(t) & 0 & \ddots & & \vdots \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & & \ddots & 0 & \hat{e}_{n-1,n}(t) - \hat{e}_{n,n-1}(t) \\ \hat{e}_{n,1}(t) - \hat{e}_{1,n}(t) & \cdots & \cdots & \hat{e}_{n,n-1}(t) - \hat{e}_{n-1,n}(t) & 0 \end{bmatrix}. \end{aligned}$$

This is the definition by Bech, Madsen and Natrop (2002), hence forth referred to as BMN. However, since $\hat{e}_{i,j}(t) - \hat{e}_{j,i}(t) = -[\hat{e}_{j,i}(t) - \hat{e}_{i,j}(t)]$, the BMN definition hides some of the finer details. This is because negative exposures do not relate to the practical application and usage of payment and netting systems, in which all netted exposures are recorded as positive amounts. Another issue with this definition is that it does not capture how, when two banks net off their exposures, the bank with the lowest exposure is reduced to zero.

BMN also define a second version of the netting operator, $N_2(*)$ as,

$$\begin{aligned}
N_2(E) &= N_2 \left(\begin{bmatrix} 0 & e_{1,2}(t) & \cdots & \cdots & e_{1,n}(t) \\ e_{2,1}(t) & 0 & \ddots & & \vdots \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & & \ddots & 0 & e_{n-1,n}(t) \\ e_{n,1}(t) & \cdots & \cdots & e_{n,n-1}(t) & 0 \end{bmatrix} \right) \\
&= \begin{bmatrix} 0 & |e_{1,2}(t) - e_{2,1}(t)| & \cdots & \cdots & |e_{1,n}(t) - e_{n,1}(t)| \\ |e_{2,1}(t) - e_{1,2}(t)| & 0 & \ddots & & \vdots \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & & \ddots & 0 & |e_{n-1,n}(t) - e_{n,n-1}(t)| \\ |e_{n,1}(t) - e_{1,n}(t)| & \cdots & \cdots & |e_{n,n-1}(t) - e_{n-1,n}(t)| & 0 \end{bmatrix}.
\end{aligned}$$

Although this definition does not have any negative exposures, it has weaknesses. Since $|e_{i,j}(t) - e_{j,i}(t)| = |e_{j,i}(t) - e_{i,j}(t)|$, this version loses much of the information because there is no way to ascertain which of the two banks has the higher exposure. This means that $N_2(*)$ only captures the greatest remaining netted amount for just one of the two banks.

In this thesis we overcome the above limitations by altering a netting definition by Fermanian and Scaillet (2003). Fermanian and Scaillet define a Netting operator $(*)^+$ as $[e_{i,j}(t), e_{j,i}(t)]^+ = \max\{e_{i,j}(t) + e_{j,i}(t), 0\}$, thus changing the above definition of $N_2(*)$ to $N_3(*)$, as shown below.

$$\begin{aligned}
N_3(E) &= N_3 \left(\begin{bmatrix} 0 & e_{1,2}(t) & \cdots & \cdots & e_{1,n}(t) \\ e_{2,1}(t) & 0 & \ddots & & \vdots \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & & \ddots & 0 & e_{n-1,n}(t) \\ e_{n,1}(t) & \cdots & \cdots & e_{n,n-1}(t) & 0 \end{bmatrix} \right) \\
&= \begin{bmatrix} 0 & \max\{e_{1,2}(t) + e_{2,1}(t), 0\} & \cdots & \cdots & \max\{e_{1,n}(t) + e_{n,1}(t), 0\} \\ \max\{e_{2,1}(t) + e_{1,2}(t), 0\} & 0 & \ddots & & \vdots \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & & \ddots & 0 & \max\{e_{n-1,n}(t) + e_{n,n-1}(t), 0\} \\ \max\{e_{n,1}(t) + e_{1,n}(t), 0\} & \cdots & \cdots & \max\{e_{n,n-1}(t) + e_{n-1,n}(t), 0\} & 0 \end{bmatrix} .
\end{aligned}$$

This definition captures the banking practice of recording the Bank with the lower exposure to have a netted exposure of zero, and also captures the requirement that banks do not have any negative netted exposures. However, this definition implicitly assumes that $e_{i,j}(t) > 0$ implies that $e_{j,i}(t) < 0$ and that $e_{i,j}(t) < 0$ implies that $e_{j,i}(t) > 0$ if netting is to occur at all, and so it requires at least one gross or pre-netted exposure to be negative. Such an approach is somewhat unorthodox since netting is a process of subtraction of positive exposure amounts.

To remedy this, the Fermanian and Scaillet definition was refined to arrive at the netting model used in this thesis. Here we use $N(*)$, where $N[e_{i,j}(t), e_{j,i}(t)] = \max\{e_{i,j}(t) - e_{j,i}(t), 0\}$ and is shown in further detail below in Equation 3.7.

$$\begin{aligned}
N(E) &= N \begin{pmatrix} 0 & e_{1,2}(t) & \cdots & \cdots & e_{1,n}(t) \\ e_{2,1}(t) & 0 & \ddots & & \vdots \\ \vdots & \ddots & \ddots & & \vdots \\ \vdots & & \ddots & 0 & e_{n-1,n}(t) \\ e_{n,1}(t) & \cdots & \cdots & e_{n,n-1}(t) & 0 \end{pmatrix} \\
&= \begin{bmatrix} 0 & \cdots & \cdots & \cdots & \max\{e_{1,n}(t) - e_{n,1}(t), 0\} \\ \max\{e_{2,1}(t) - e_{1,2}(t), 0\} & \ddots & & & \vdots \\ \vdots & \ddots & \ddots & & \vdots \\ \vdots & & \ddots & 0 & \max\{e_{n-1,n}(t) - e_{n,n-1}(t), 0\} \\ \max\{e_{n,1}(t) - e_{1,n}(t), 0\} & \cdots & \max\{e_{n,n-1}(t) - e_{n-1,n}(t), 0\} & 0 & 0 \end{bmatrix}.
\end{aligned} \tag{3.7}$$

We can now demonstrate how this model can be applied to data by presenting an example of two Banks. An example of three Banks together with other formulaic netting results that have been derived as part of this thesis can be found in the **Appendix - Additional Theoretical Results**.

3.4.2 Example of Two Banks

Take two Banks x_1 and x_2 . Although a diagram is not required for this case, such diagrams will be useful for three or more Banks. Their exposures, through bilateral netting agreements can be shown schematically in Figure 3.2.

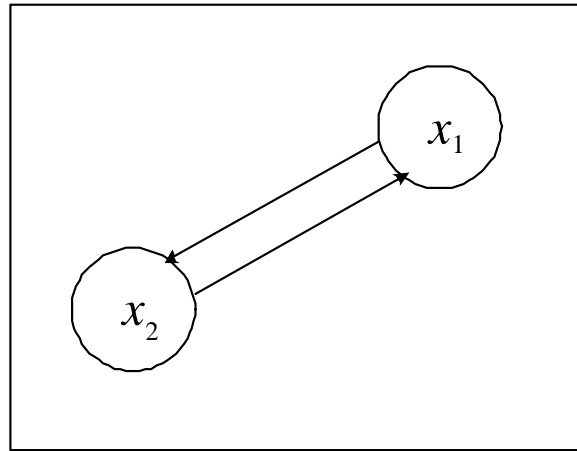


Figure 3.2: Exposures Between Two Banks

Suppose that in our two Bank example of Bank x_1 and Bank x_2 , that at some point in time t , $e_{1,2}(t) = \$60,000,000$ and $e_{2,1}(t) = \$65,000,000$. Following on from our netting formulation of Equation 3.7, the number of Banks n is $n = 2$, giving,

Without Netting

$$\begin{aligned}
 E &= \begin{bmatrix} 0 & e_{1,2}(t) \\ e_{2,1}(t) & 0 \end{bmatrix} \\
 &= \begin{bmatrix} 0 & \$60,000,000 \\ \$65,000,000 & 0 \end{bmatrix}.
 \end{aligned}$$

With Netting

$$\begin{aligned}
N(E) &= N \left(\begin{bmatrix} 0 & e_{1,2}(t) \\ e_{2,1}(t) & 0 \end{bmatrix} \right) \\
&= \begin{bmatrix} 0 & \max\{e_{1,2}(t) - e_{2,1}(t), 0\} \\ \max\{e_{2,1}(t) - e_{1,2}(t), 0\} & 0 \end{bmatrix} \\
&= \begin{bmatrix} 0 & \max\{\$60,000,000 - \$65,000,000, 0\} \\ \max\{\$65,000,000 - \$60,000,000, 0\} & 0 \end{bmatrix} \\
&= \begin{bmatrix} 0 & 0 \\ \$5,000,000 & 0 \end{bmatrix}.
\end{aligned}$$

Hence, under netting, Bank x_1 no longer has an exposure to Bank x_2 , whilst Bank x_2 has reduced its exposure to Bank x_1 down to \$5,000,000.

3.5 MC Simulation Methodology

The MC simulation method is widely applied to large and complex problems to obtain approximate solutions. This method has been successfully applied to problems in physical sciences and, more recently, in finance. Many difficult financial engineering problems such as the valuation of multidimensional and path-dependent options contracts can be approximated due to this technique. Of particular importance to this thesis is its application to the simulation and modelling of the stochastic volatility of FX rates.

To a certain extent, MC simulation is not strictly required here due to the nature of GARCH itself. The finer aspect of GARCH reveals that the way in which the GARCH parameters are determined is via an optimisation of ordinary least squares (OLS) errors. This means that every time we fit a GARCH model, we are essentially generating multiple estimates, checking convergence and then refining the estimates, until the differences are minimised and that the process has converged. This approach is a numerical approximation technique that is very similar to

MC simulation. These iterations were carried out using the S+PLUS module S+FinMetrics.

MC simulation works by passing random numbers of known characteristics into the model under consideration. One then analyses the resulting distribution to determine the overall effect of the model. Convergence analysis of the simulations is then undertaken to determine whether a stable solution or value can be determined. In practice, the random numbers are generated from mathematical formulas and are hence called pseudo-random numbers. In our framework this translates into the pseudo code of Algorithm 1 below.

Algorithm 1: MC Simulation of FX Rates

```

Generate a suitable distribution of pseudo-random numbers
For j=1 to N
    Take j-th pseudo-random number and pass as input to the model
    Measure model output
    Determine nature of model output distribution
    If nature of the distribution has converged Then
        exit loop
    Else
        j=j+1
    EndIf
End for

```

For the MC algorithm to be applied effectively, at least the following three main conditions must be satisfied. First, the random number dataset must be sufficiently large enough to ensure proper stress testing of the model (Shapiro and Homem-de-Mello, 2000). Second, the random number dataset must be sufficiently random so as to not display banding or clustering (Marsaglia, 1968). Third, the pseudo-random number generator must generate sequences of sufficiently large enough period to allow accurate modelling (Brent and Zimmermann, 2003). Currently, the Mersene Twister algorithm is the most popular and powerful generator with a period of $2^{19,937} - 1$ (Matsumoto and Nishimura, 1998).

To apply a MC simulation algorithm and uphold the above conditions, we need to review the literature on how one can measure progress towards convergence around an approximate

solution that is sufficiently accurate for one's needs.

3.5.1 MC Expected Error and Estimates

MC Simulation is an attractive estimation procedure for GARCH because the conditional distribution of the latent variable is straight forward to derive given the GARCH coefficients. In turn, the conditional distributions of the GARCH coefficients typically are Gaussian, given values for the latent variable. The key idea behind MC estimation is that after a sufficient number of iterations, the draws from the respective conditional distributions jointly represent a draw from the joint posterior distribution, which often cannot be evaluated directly (Gelfand and Smith, 1990).

Consider a function $h(x)$ where x is an n -dimensional vector over some volume Ω .

$$\int h(x)dx = \lambda(\Omega) \int h d\phi = \lambda(\Omega)E[h], \quad (3.8)$$

where $d\phi = dx/\lambda(\Omega)$ and $\lambda(\Omega)$ is the volume of Ω , i.e. its Lebesgue measure. Note that h need not be non-negative or positive (Al-Mharmah, 1998). $E[h]$ is the expected value in integrating the function $h(x)$ and is typically difficult to express in a closed analytic form. MC simulation can form an estimate for $E[h]$, which has the following form,

$$E[h] = \frac{1}{N} \sum_{i=1}^N f(x_i) + \epsilon, \quad (3.9)$$

where ϵ is an error term and $f(x)$ is the function that can't be integrated easily. $E[h]$ becomes an approximation when we assume that ϵ is zero. What makes it MC simulation is when the points x_i are distributed according to the distribution ϕ , the variance of the distribution is defined as follows,

$$\sigma^2(h) = E \left[(h - E(h))^2 \right]. \quad (3.10)$$

The expected error of the MC estimate is thus defined to be its sample variance, i.e.,

$$\sigma_\epsilon^2(h) = E \left[\left(\frac{1}{N} \sum_{i=1}^N f(x_i) - E(h) \right)^2 \right] = \frac{\sigma^2(h)}{N}, \quad (3.11)$$

(Niederreiter, 1992). Central to these analyses is often the normal or Gaussian Distribution, as the most common distribution in nature.

We can now begin relating these measures back to finance, by addressing the impact of netting on a portfolio of returns that are dependent on FX rate fluctuations. However, as we try to estimate a portfolio loss distribution measure such as VaR via MC Simulation, we are faced with a number of financial issues. These issues are listed below and to overcome these, certain extensions need to be made to MC Simulation.

3.5.2 Extensions of MC Simulation for Financial Time Series Data

The experience of decades of quantitative analysis of financial data has shown some specific ways in which observed financial time series can fail to satisfy the properties of the Gaussian distribution. The reasons for this are;

1. Fat tails: the marginal distributions of financial time series have a typical shape which differs from the Gaussian. Many financial time series are higher in the tail and central areas, while lower in the shoulders, characteristic of the leptokurtic nature of financial returns.
2. Stochastic volatility: it is observed (and expected by no arbitrage considerations) that the autocorrelation of log returns decays quickly to near zero over the time scale of minutes. However, higher order autocorrelations exhibit memory effects which can be interpreted as serial correlation in the covariance matrix.
3. Skewness: left tails tend to be slightly fatter than right tails.
4. Scaling effects: marginal distributions of asset returns measured over two different time increments are related by a simple scaling transformation (Gopikrishnan, *et. al.*, 1999).
5. Multivariate effects: observed multivariate data has “tail dependence” (Joe, 1997). The observed tail dependence corresponds to the econometric statement that joint extreme moves are systematically more frequent than is consistent with multivariate Gaussian models.

The final effect listed above has important implications for portfolio theory. It implies that the strategy of portfolio diversification, the most important principle in financial risk management, has less power than otherwise expected to mitigate risk under scenarios of market distress (Abad and Hurd, 2003). This means that under MC simulation of GARCH(1,1) models, alternative

distributions may most likely be required for at least some of the FX rates. The typical extensions or refinements to the Gaussian distribution are the Student-t, generalised error distribution (GED) and the double error distribution (DED).

In summary, we have investigated the FX daily rates of five international currencies and have examined their stochastic volatility. By having then discussed the GARCH methodology, we have been able to generate five realistic time series models. These models would ultimately feed into the netting model, which was also detailed. Finally, we discussed the MC simulation methodology which explained how the models will generate the simulated FX rates through the use of appropriate pseudo-random numbers. We are now ready to apply these methodologies to derive the results.

Chapter 4

Modelling Results

After having extensively detailed the modelling methodology, we are now ready to review the results. This chapter will be composed of the following sections; FX Rate Time Series Data, GARCH Models, MC GARCH Simulations, and Application of Netting Model. This essentially allows for the input data to the model to be analysed, the model(s) generalised and simulated, and finally for the netted distributions to be compared.

4.1 FX Rate Time Series Data

We begin our modelling by analysing the FX Daily Average Interbank Rate for the five currencies we have chosen. The data was obtained from OANDA Corporation ¹. Figure 4.1 below charts the nature of this data through the use of S-PLUS and its financial package S+FinMetrics. As mentioned above, all analysis was carried out in this environment.

¹OANDA Corporation provides data at <http://www.oanda.com/convert/fxhistory>.

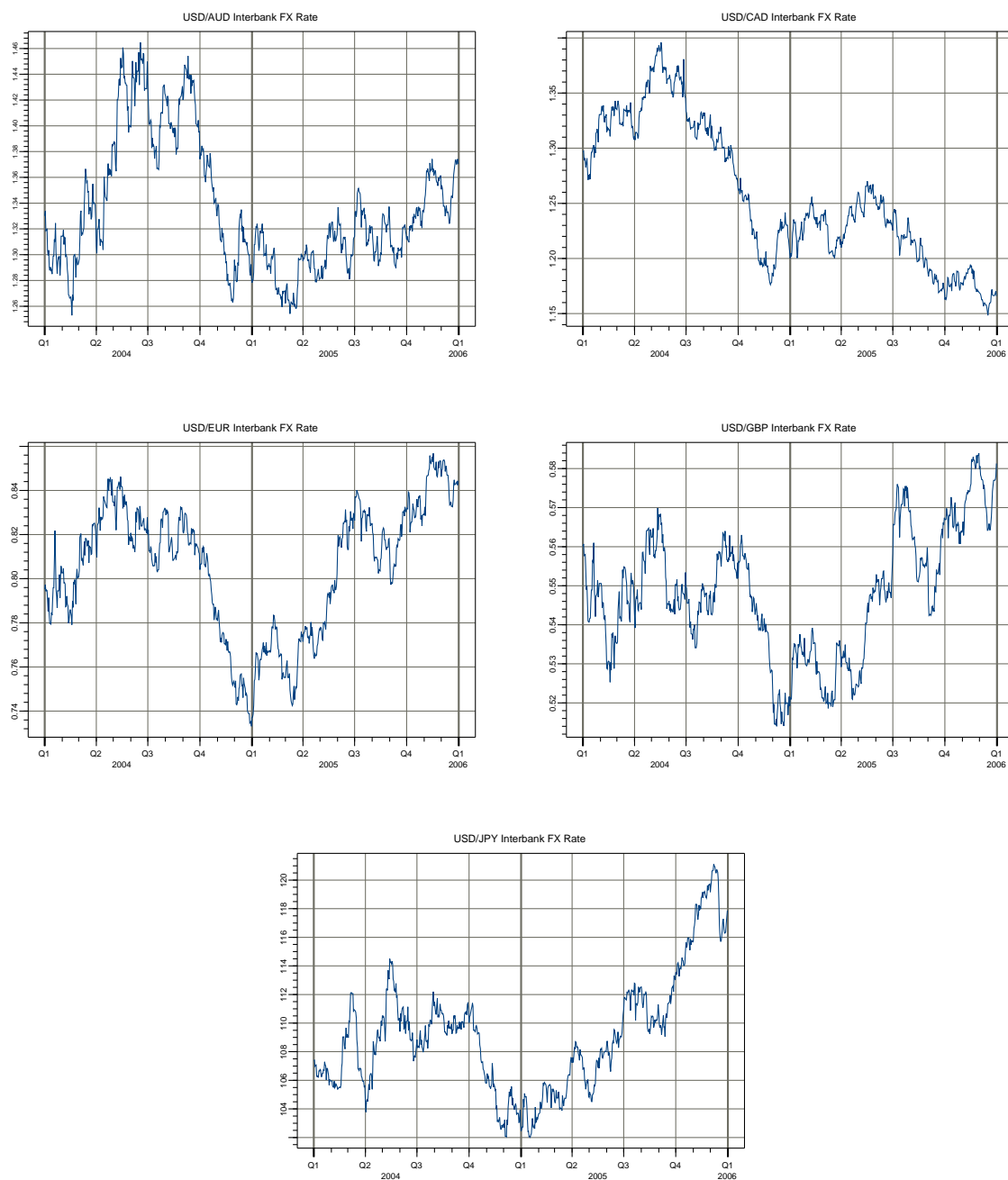


Figure 4.1: Daily FX Rate Data for 2004-2005

Figure 4.1 shows two years worth of data. Since we will be modelling the first year, Figure 4.2 shows just the first year in more detail.

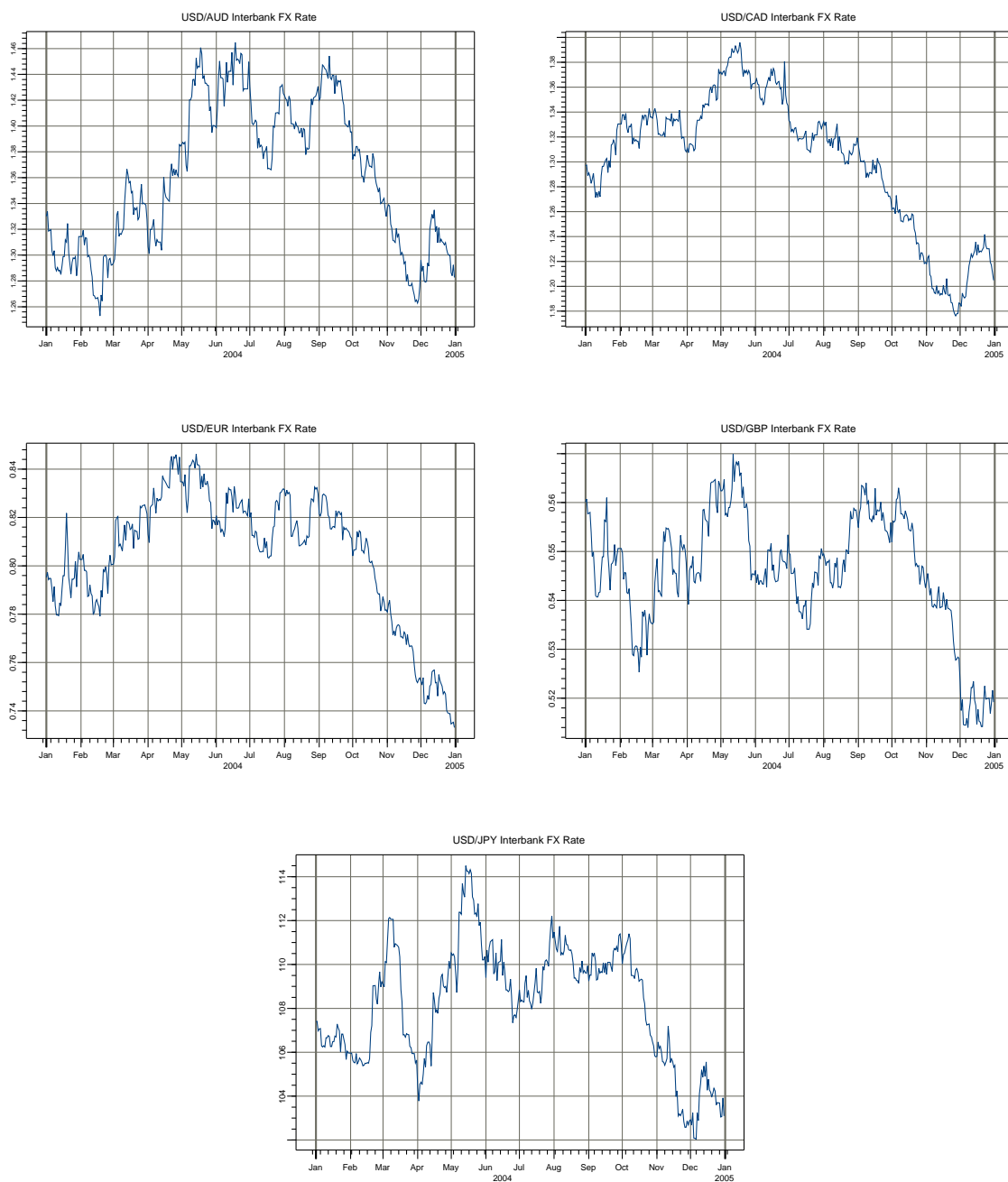


Figure 4.2: Daily FX Rate Data for 2004

Since GARCH models the volatility of the differences and not the original time series data, daily differences were calculated for Figure 4.1 as shown in Figure 4.3, and calculated for Figure 4.2 as shown in Figure 4.4.

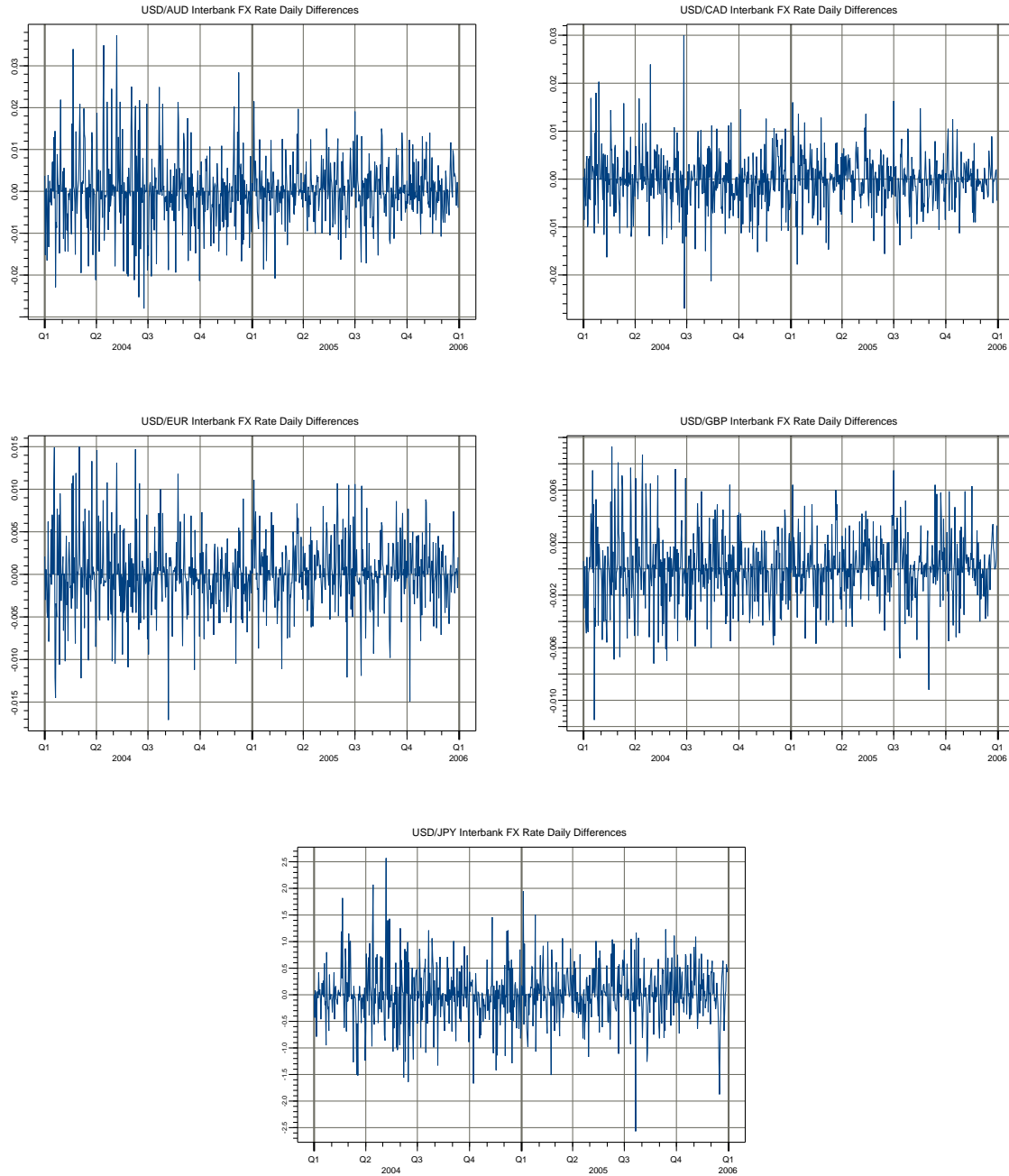


Figure 4.3: Daily FX Rate Differences for 2004-2005

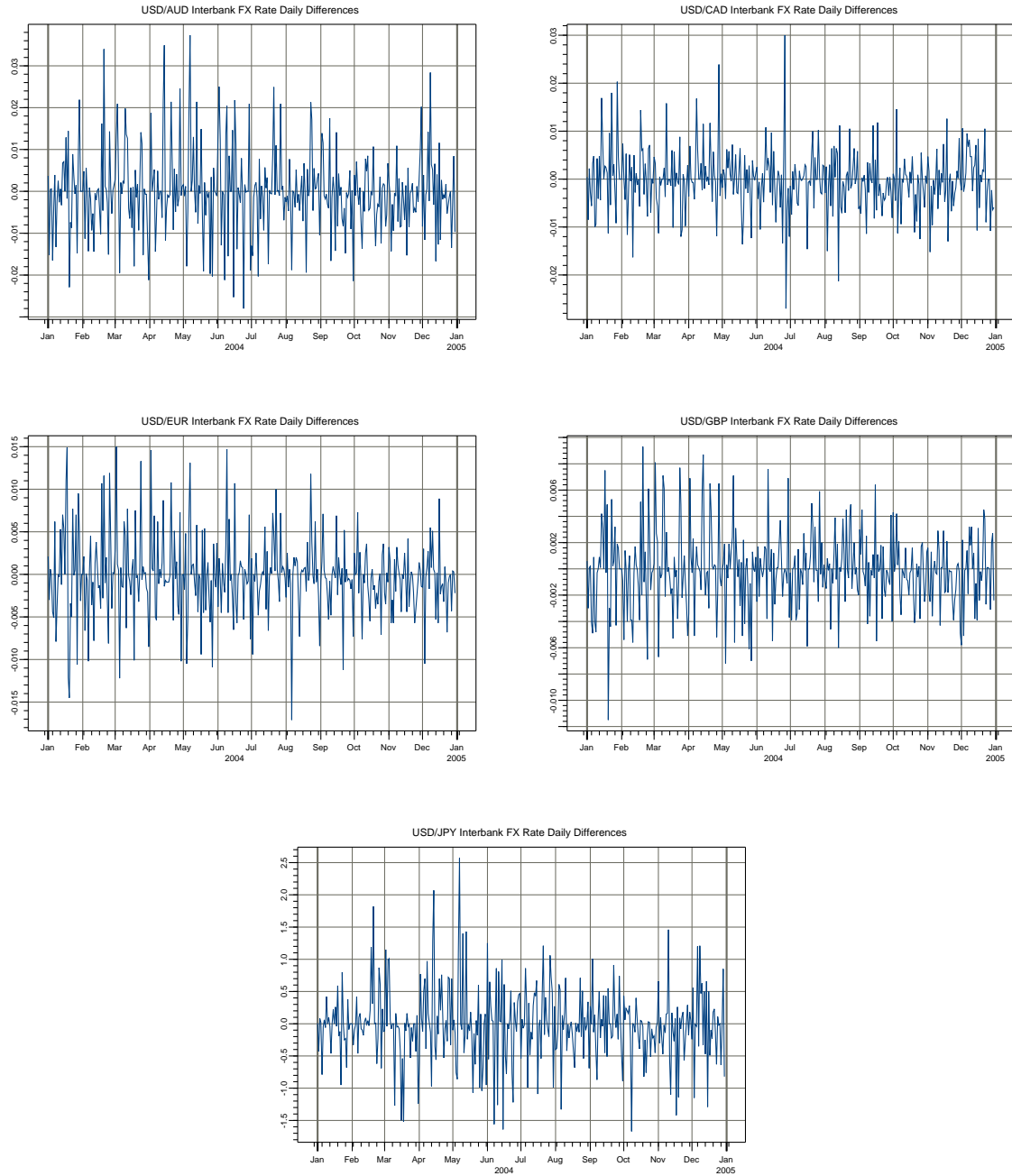
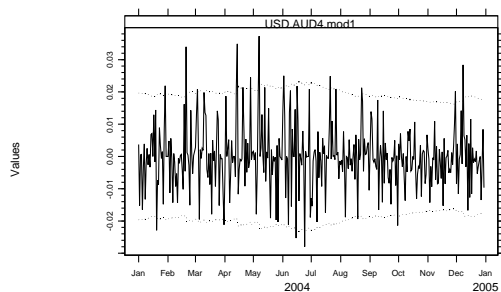


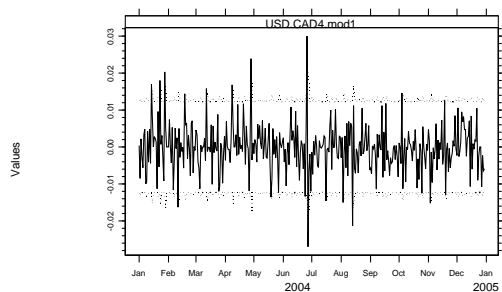
Figure 4.4: Daily FX Rate Differences for 2004

Since we will be using the first year's difference data to build each model, we refer to 4.4. This shows the volatility of each FX rate's differences, and suggests that they are generally bound above by 0.01 and below by -0.01. To obtain a clearer measure for these bounds, the differences were plotted with two conditional standard deviations superimposed in Figure 4.5.

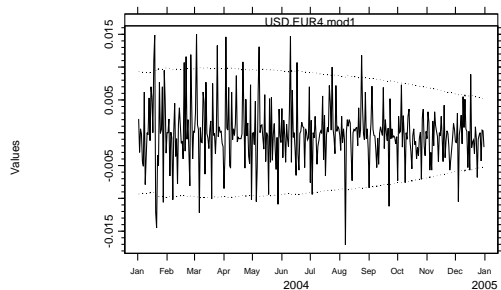
USD/AUD FX Rate Series with 2 Conditional SD Superimposed



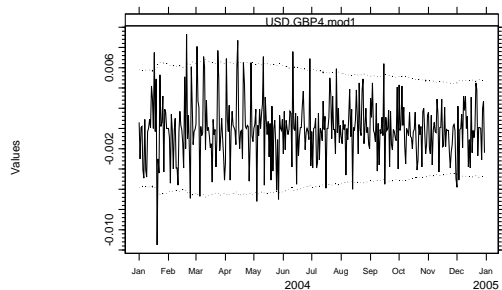
USD/CAD FX Rate Series with 2 Conditional SD Superimposed



USD/EUR FX Rate Series with 2 Conditional SD Superimposed



USD/GBP FX Rate Series with 2 Conditional SD Superimposed



USD/JPY FX Rate Series with 2 Conditional SD Superimposed

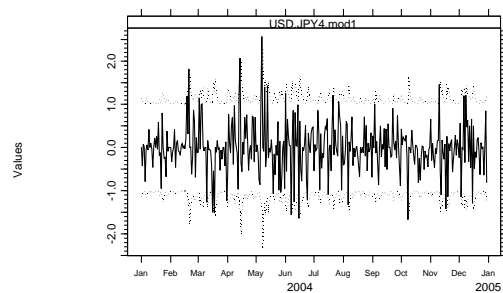


Figure 4.5: FX Daily Differences and Two Conditional Standard Deviations Superimposed for 2004

The observed patterns may be due to autocorrelation or, even more likely, heteroskedasticity and so auto correlation function (ACF) plots of the FX rate daily differences were graphed in Figure 4.6 together with ACF plots of the squared FX rate daily differences were graphed in Figure 4.7.

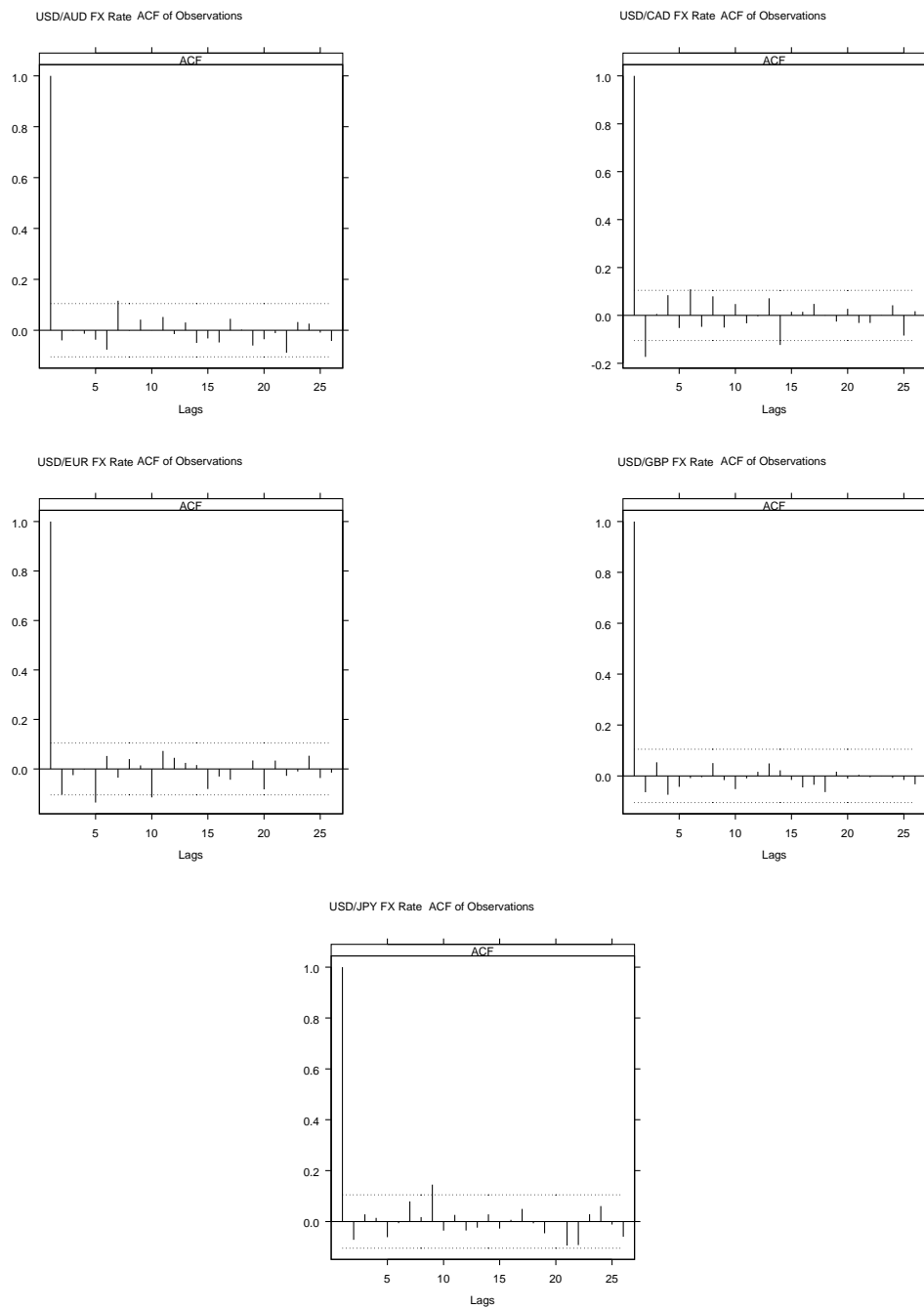
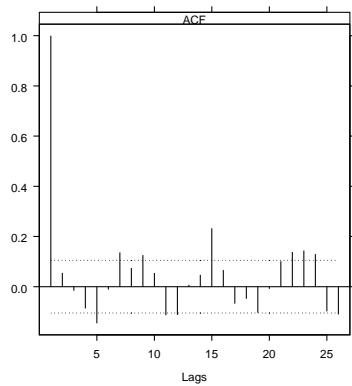
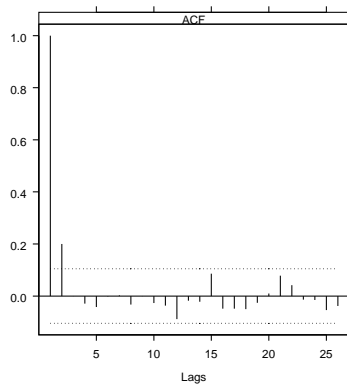


Figure 4.6: ACF of FX Rate Daily Differences for 2004

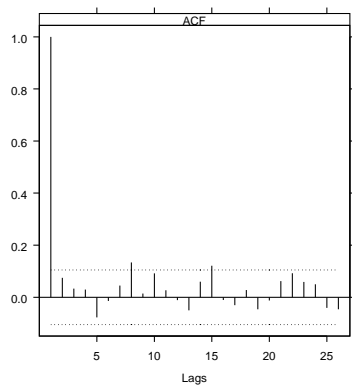
USD/AUD FX Rate ACF of Squared Observations



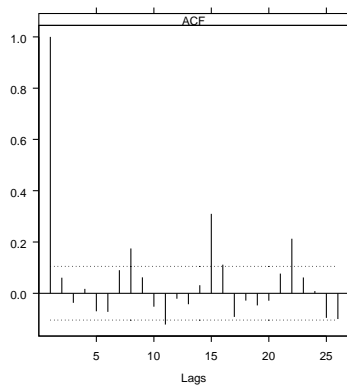
USD/CAD FX Rate ACF of Squared Observations



USD/EUR FX Rate ACF of Squared Observations



USD/GBP FX Rate ACF of Squared Observations



USD/JPY FX Rate ACF of Squared Observations

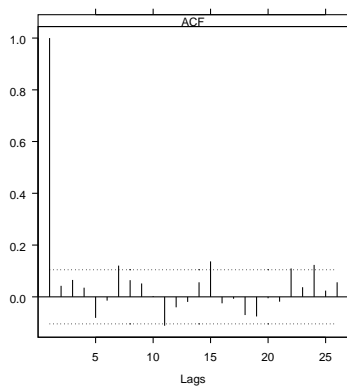


Figure 4.7: ACF of FX Rate Squared Daily Differences for 2004

From Figure 4.6 and Figure 4.7, both the ACF of the differences and the squared differences exhibit some autocorrelation, up to at least lag 25. Since the squared differences measure the second order moment of the original differences, this result indicates that the variance of these differences that are conditional on their past history may change over time, or equivalently, these differences may exhibit time varying conditional heteroskedasticity or volatility clustering. To determine whether this time dependence was persistent, a cross correlation function (CCF) plot was graphed in Figure 4.8 between the squared daily differences, crossed with just the daily differences.

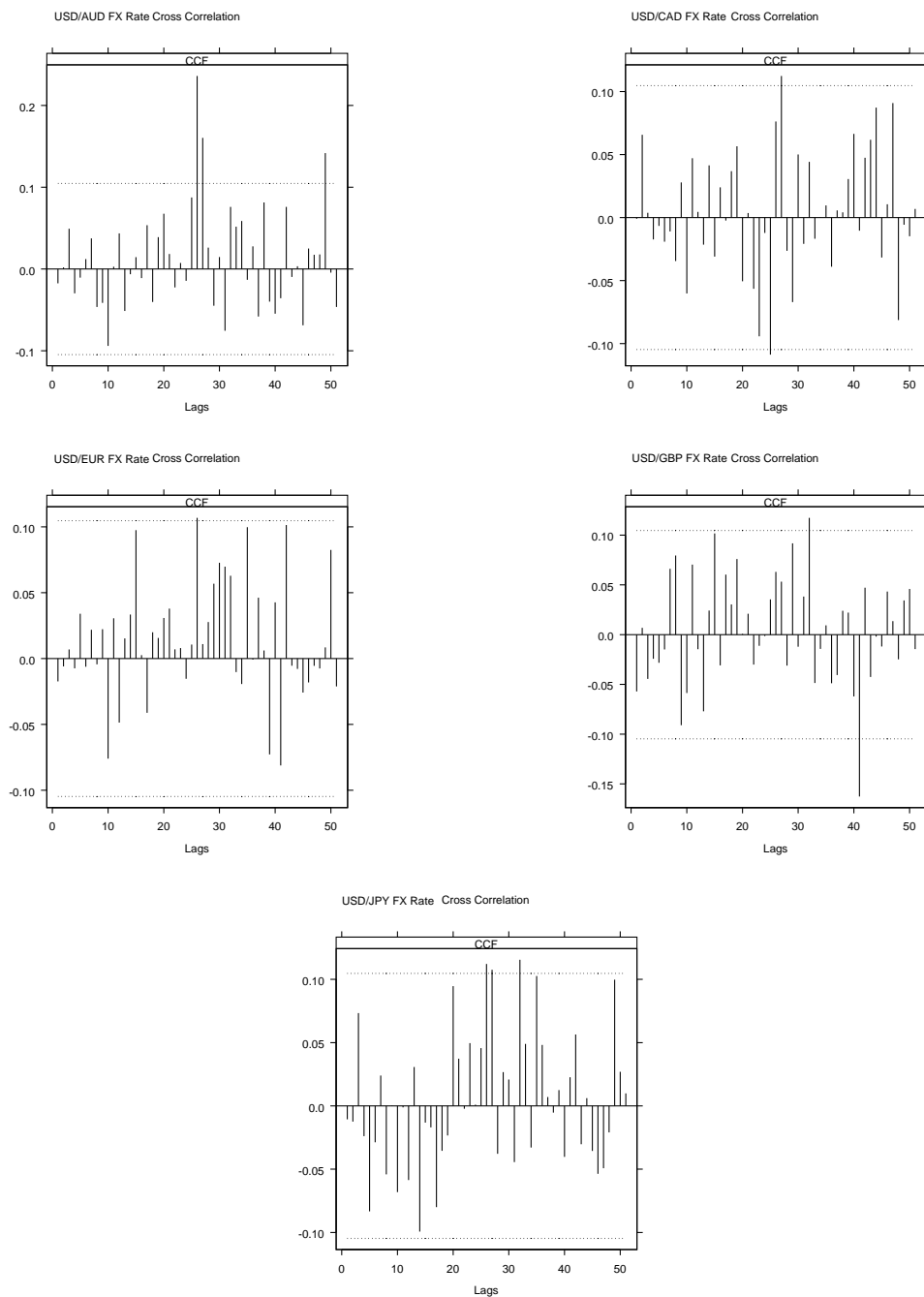


Figure 4.8: FX Rate Squared Daily Differences Cross Correlation with Daily Differences for 2004

Figure 4.8 doesn't show any obvious CCF characteristics over time. This provided further support to use a GARCH framework. To shed further light into this figure, histograms were graphed in Figure 4.9.

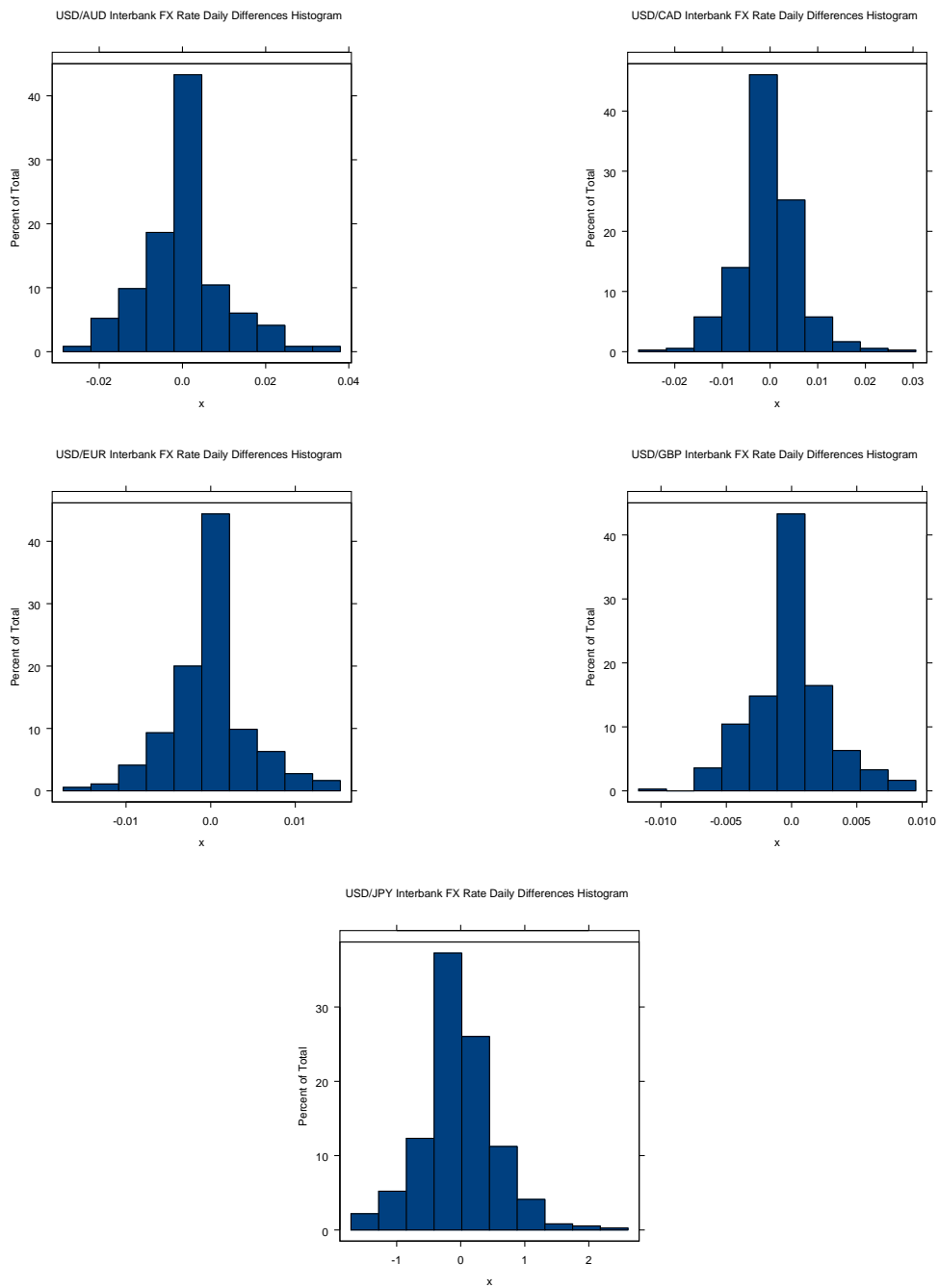


Figure 4.9: FX Rate Differences Histograms for 2004

Figure 4.9 suggests that the differences for some but not all FX rates may not be normally distributed. To further explore this hypothesis, a quantile (QQ) plot was graphed in Figure 4.10.

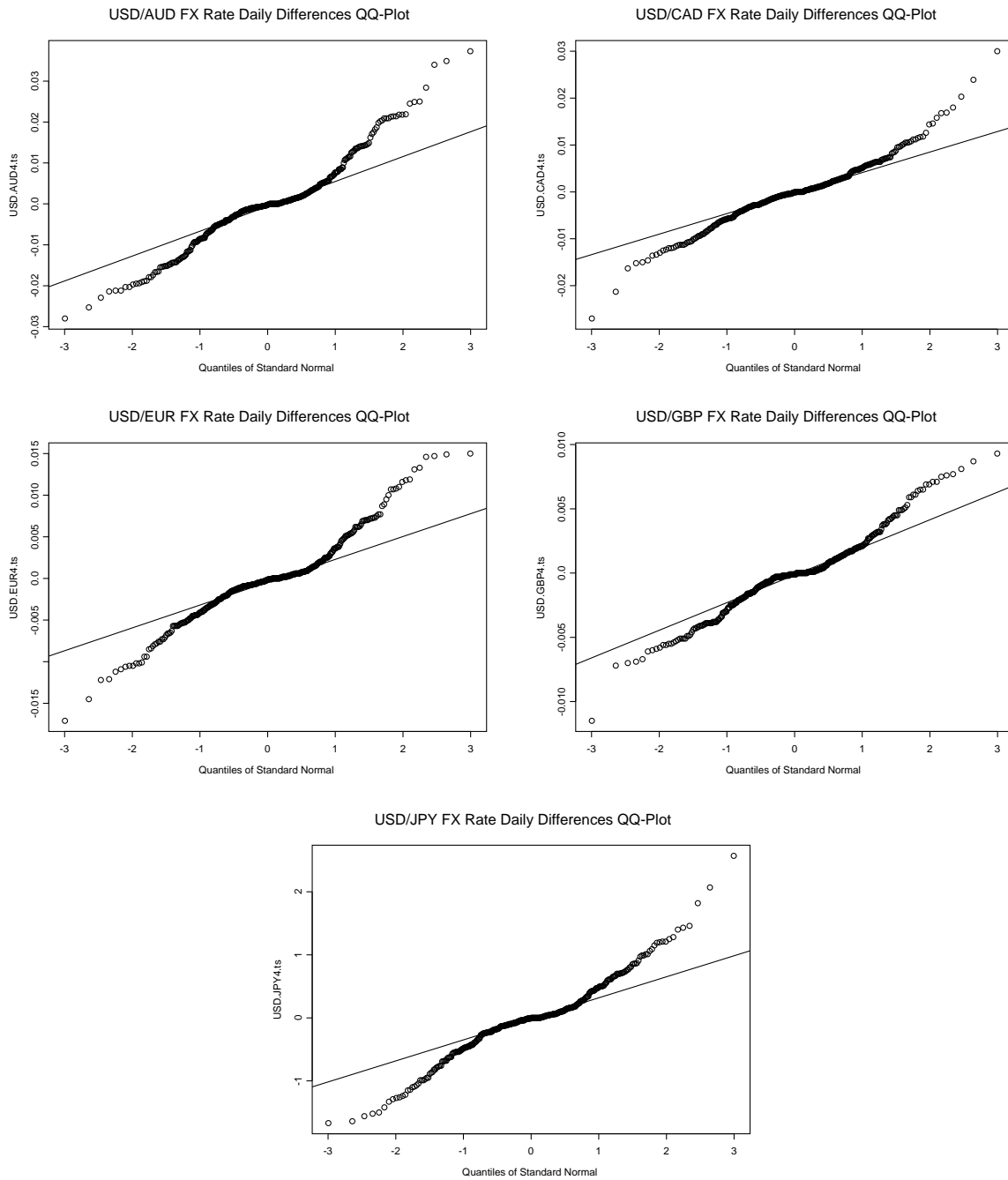


Figure 4.10: FX Rate Differences QQ-Plots for 2004

Figure 4.10 provides further evidence that the differences for some but not all FX rates may not be normally distributed, but we cannot as yet determine which is a more suitable distribution. Finally, the summary statistics are listed in Table 4.1.

Table 4.1: FX Rate Summary Statistics for 2004

FX Rate	Mean	Variance	S.D.	Skewness	Kurtosis
USD/AUD	-0.0001299	0.0000969	0.0098415	0.4627727	1.446223
USD/CAD	-0.0002526	0.0000413	0.0064256	0.2439137	2.712788
USD/EUR	-0.0001701	0.0000219	0.0046760	0.2787559	1.745698
USD/GBP	-0.0001121	0.0000085	0.0029195	0.1887077	1.150171
USD/JPY	-0.0117810	0.3257136	0.5707132	0.2659795	2.035926

From Table 4.1, four of the five currencies are similarly distributed, except for USD/JPY which has a much larger variance/standard deviation.

4.2 GARCH Models

To further verify whether a GARCH specification was appropriate, the null hypothesis was proposed that the data had no ARCH effects. The ARCH test in S+FinMetrics is essentially a Lagrange Multiplier Test and resulted in Table 4.2.

Table 4.2: FX Rate ARCH Test Summary Statistics for 2004

FX Rate	Test Stat	p-Value	D.F.
USD/AUD	51.3305	0.0015	25
USD/CAD	31.5985	0.1700	25
USD/EUR	37.2604	0.0545	25
USD/GBP	69.5601	0.0000	25
USD/JPY	36.8337	0.0599	25

Since the p-values are all smaller than the conventional 5% level, we accept the alternative hypothesis (i.e. that there are ARCH effects) and we proceed to model GARCH effects for each FX rate. Since ARCH effects were present, ordinary least squares (OLS) would not suffice and the more generalised maximum likelihood estimation (MLE) technique would be required for parameter estimation.

The GARCH algorithm ² in S+FinMetrics was executed for each FX rate and the iterations all converged except for the USD/EUR rate. For the USD/EUR rate, no local maximum was reached in a reasonable number of iterations. This suggests, although further analysis is required, and that although a GARCH(1,1) model is appropriate, a Gaussian error distribution

²S+FinMetrics uses the BHHH Algorithm (Berndt, Hall, Hall and Hausman, 1974).

is not the best possible choice. This also suggested that the estimated asymptotic variance is not well defined under such a distribution. Figure 4.11 then was graphed to assess the overall performance of the GARCH(1,1) models at predicting the volatility.

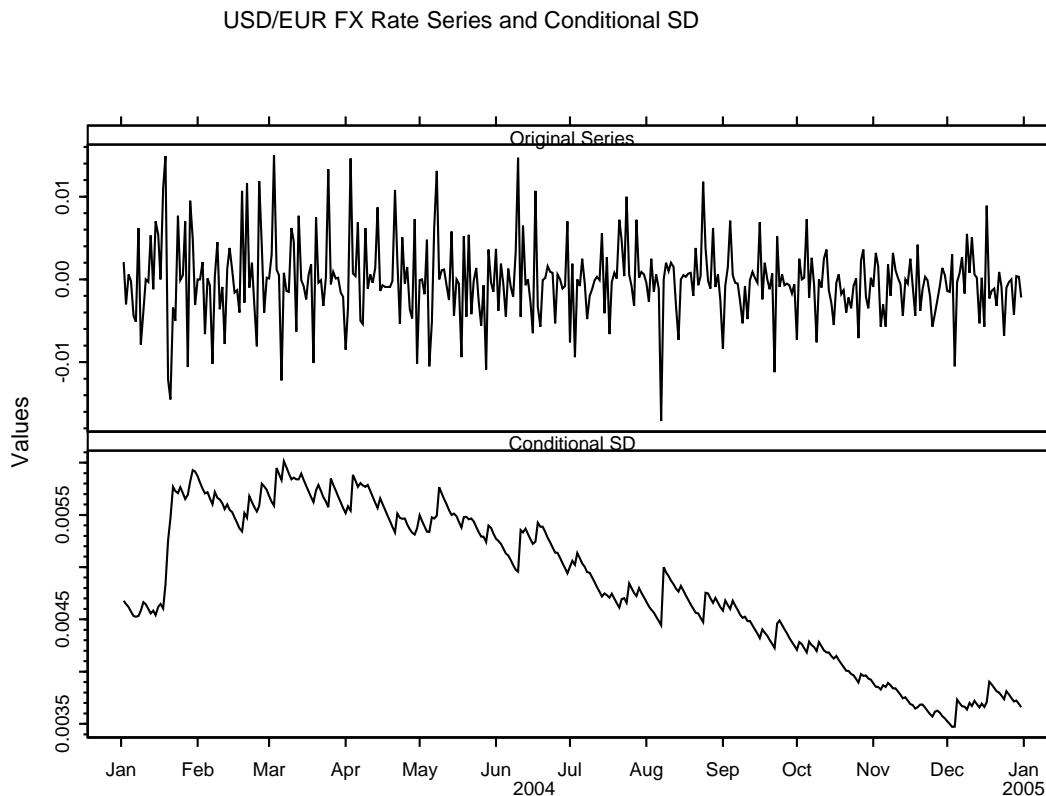


Figure 4.11: USD/EUR FX Daily Differences and GARCH(1,1) Conditional Standard Deviation for 2004 Using Student-t Distribution

Figure 4.11 shows that for the 3rd panel (i.e. USD/EUR) the model's volatility decays consistently over time. This is because $\left(\sum_{i=1}^q a_i + \sum_{j=1}^p b_j\right) > 1$ and that the GARCH(1,1) model was not correctly specified. This is further evidence that the Gaussian distribution was not appropriate for the USD/EUR GARCH(1,1). A number of underlying non-Gaussian error distributions were then examined, namely the Student-t Distribution, the GED and the DED. The corresponding GARCH(1,1) summary statistics are shown in Table 4.3 ³.

³Table 4.3 looks at GARCH(1,1) model's parameters in relation to each of the four main distributions. As will be made more apparent in Equation 4.1, through the notation of S+FinMtrics, C refers to c , A refers to a_0 , ARCH(1) refers to a_1 , and GARCH(1) refers to b_1 .

Table 4.3: Distribution Specifications for USD/EUR

Distribution	Parameter	Value	Std.Error	t-Value	Pr(> t)
Gaussian	C	-0.0003454	0.0001987	-1.738	0.0415
	A	-0.0000001	0.0000000	-3.294	0.0005
	ARCH(1)	0.0052270	0.0035010	1.493	0.0682
	GARCH(1)	0.9986000	0.0055550	179.759	0.0000
Student-t	C	-0.0003773	0.0001903	-1.983	0.0241
	A	0.0000001	0.0000002	0.235	0.4072
	ARCH(1)	0.0201200	0.0182700	1.102	0.1357
	GARCH(1)	0.9783000	0.0207700	47.101	0.0000
GED	C	-0.0001000	0.0000701	-1.428	0.0771
	A	0.0000001	0.0000003	0.222	0.4122
	ARCH(1)	0.0165800	0.0193000	0.859	0.1954
	GARCH(1)	0.9793000	0.0288000	34.007	0.0000
DED	C	-0.0002000	0.0001531	-1.307	0.0961
	A	-0.0000000	0.0000002	-0.111	0.4558
	ARCH(1)	0.0102700	0.0114600	0.897	0.1853
	GARCH(1)	0.9872000	0.0189800	52.009	0.0000

Table 4.3 shows that for the constant C, the Student-t was the distribution with the lowest $Pr(> ||t||)$ and t-value. This together with the fact that the Student-t is a common alternative to the Gaussian (Bollerslev, 1986), made it the overall better model fit to the USD/EUR FX rate data. The final models were then shown in Figure 4.12 together with their daily differences.

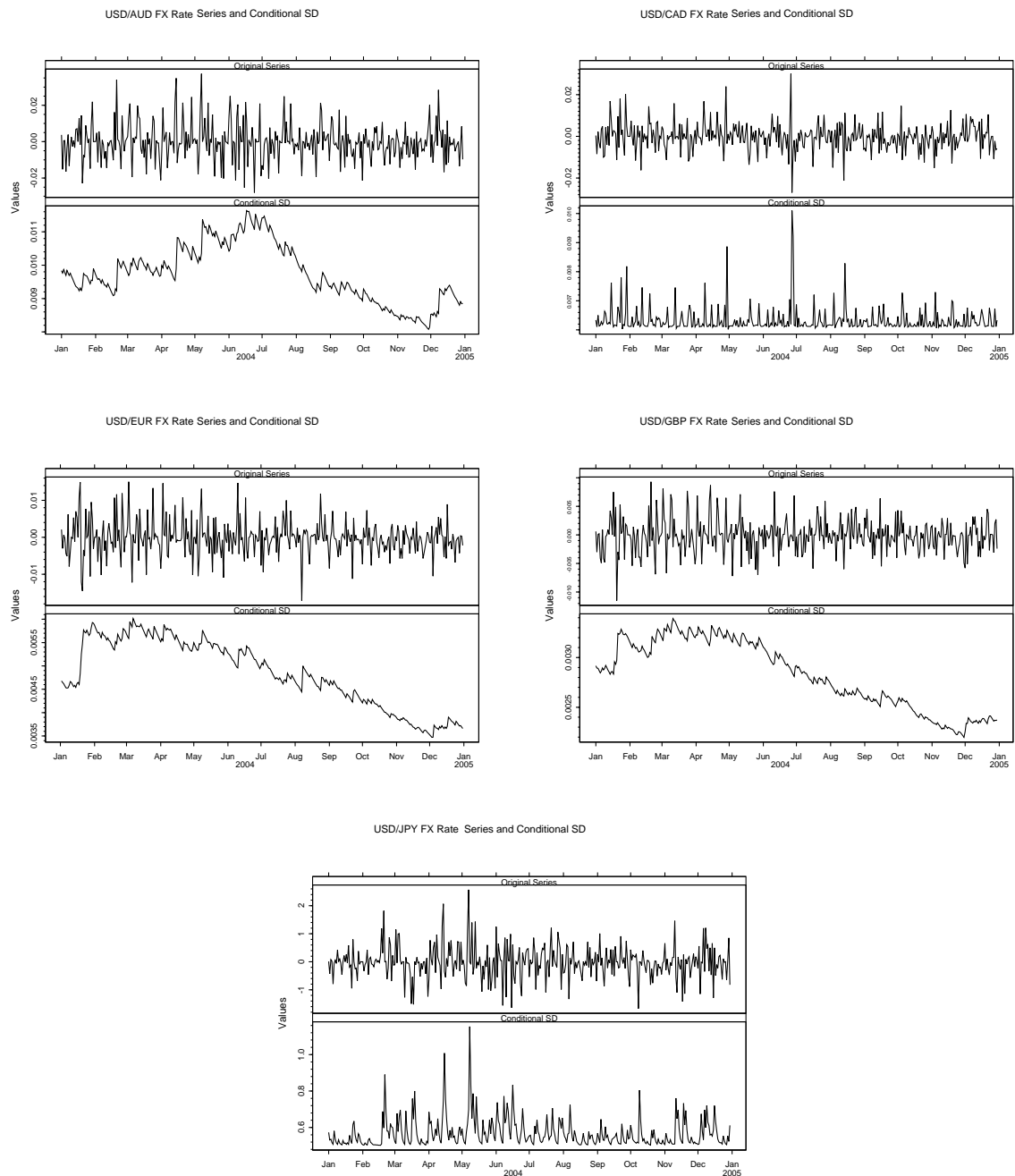


Figure 4.12: FX Daily Differences and GARCH(1,1) Conditional Standard Deviation for 2004

To further verify this choice of the Student-t, Figure 4.12 was magnified for the USD/EUR FX rate in Figure 4.13.

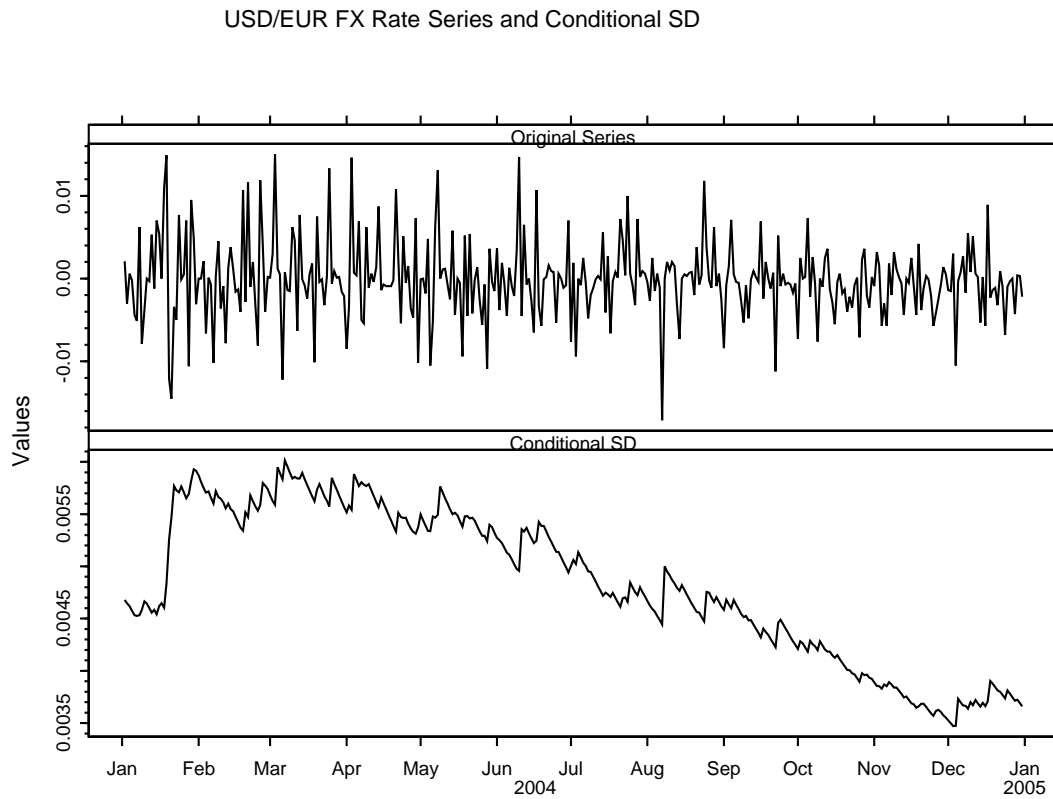


Figure 4.13: USD/EUR FX Daily Differences and GARCH(1,1) Conditional Standard Deviation for 2004 Using Student-t Distribution

Figure 4.13 further justifies the use of Student-t by highlighting that the conditional standard deviation is not as monotonic as for the use of the Gaussian. Figure 4.14 was then graphed to help assess model fit and the resulting residuals.

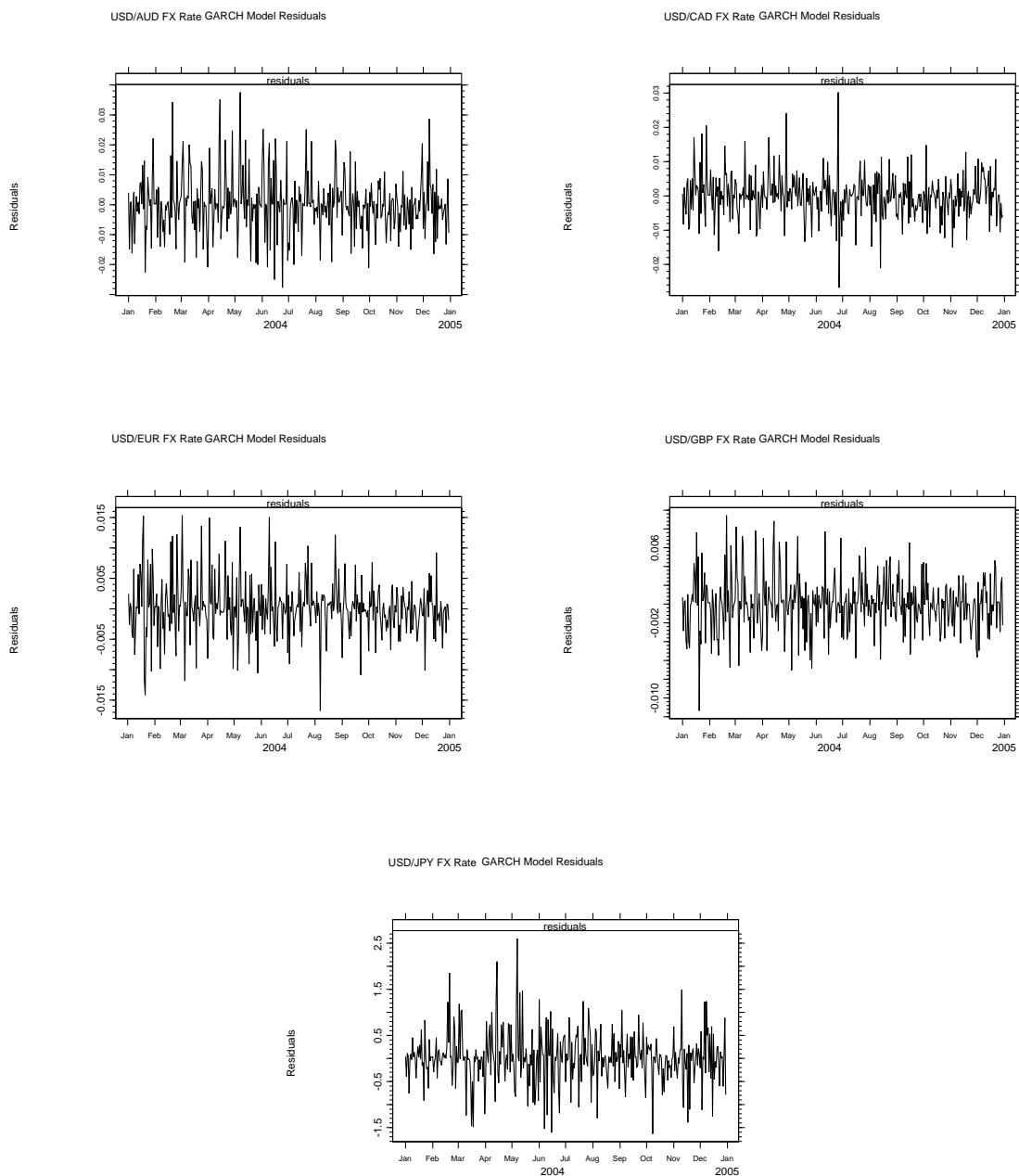


Figure 4.14: FX GARCH(1,1) Residuals for 2004

Figure 4.14 shows that the GARCH(1,1) residuals are behaving as expected and follow the overall pattern of the daily difference volatility. To obtain further insight into this GARCH modelling, the residuals were standardised to produce Figure 4.15.

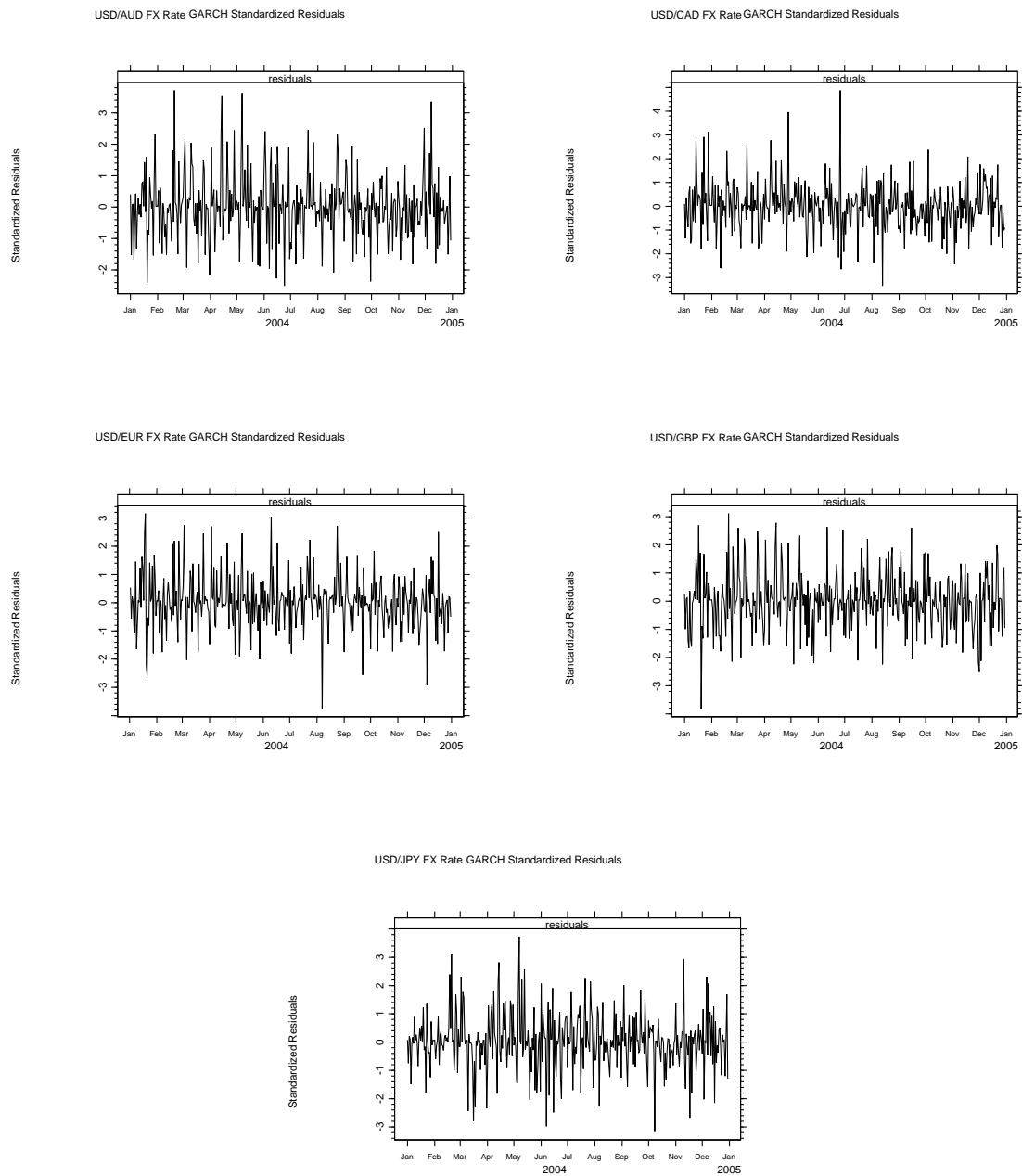


Figure 4.15: FX GARCH(1,1) Standardised Residuals for 2004

Figure 4.15 shows that these standardised residuals tend to mainly vary within ± 1 . Furthermore, the ACF of these standardised residuals was graphed in Figure 4.16.

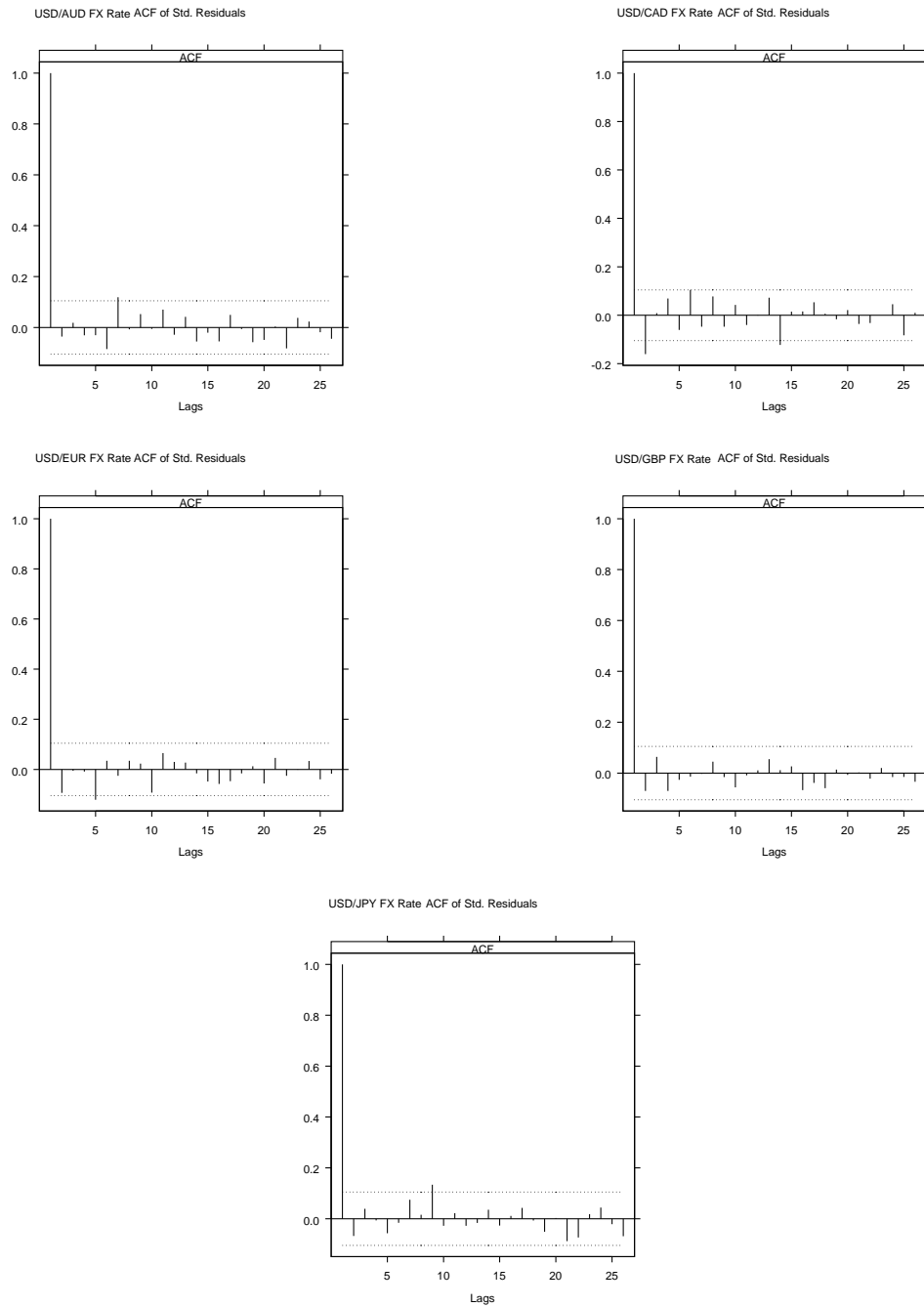
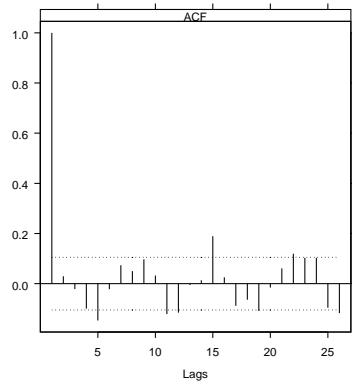


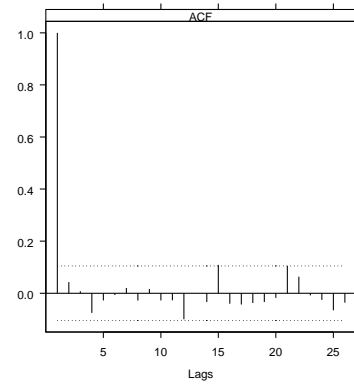
Figure 4.16: ACF of Standardised FX GARCH(1,1) Residuals for 2004

Figure 4.16 shows that the standardised residuals have the similar ACF plots as the original differences of Figure 4.4, exhibiting declining lags. To further complete the picture, the ACF of the squared standardised residuals was plotted in Figure 4.17.

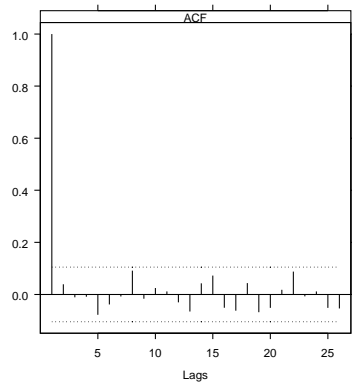
USD/AUD FX Rate ACF of Squared Std. Residuals



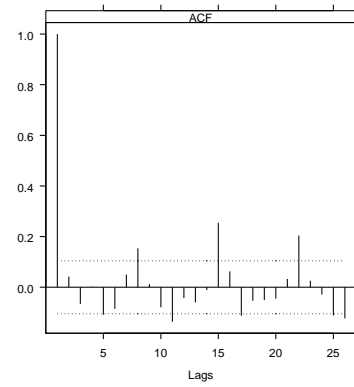
USD/CAD FX Rate ACF of Squared Std. Residuals



USD/EUR FX Rate ACF of Squared Std. Residuals



USD/GBP FX Rate ACF of Squared Std. Residuals



USD/JPY FX Rate ACF of Squared Std. Residuals

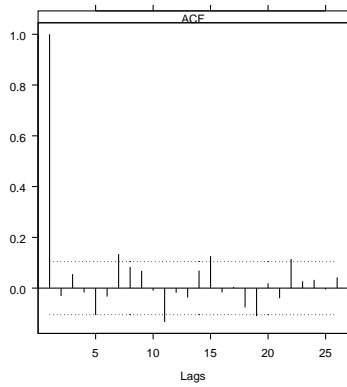


Figure 4.17: ACF of Squared Standardised FX GARCH(1,1) Residuals for 2004

Figure 4.17 shows that the squared standardised residuals have some serial correlation as evidenced by the persistent oscillation between positive and negative values for all FX rates. To shed further light into the ACF plots, a cross correlation function (CCF) plot was graphed in Figure 4.18.

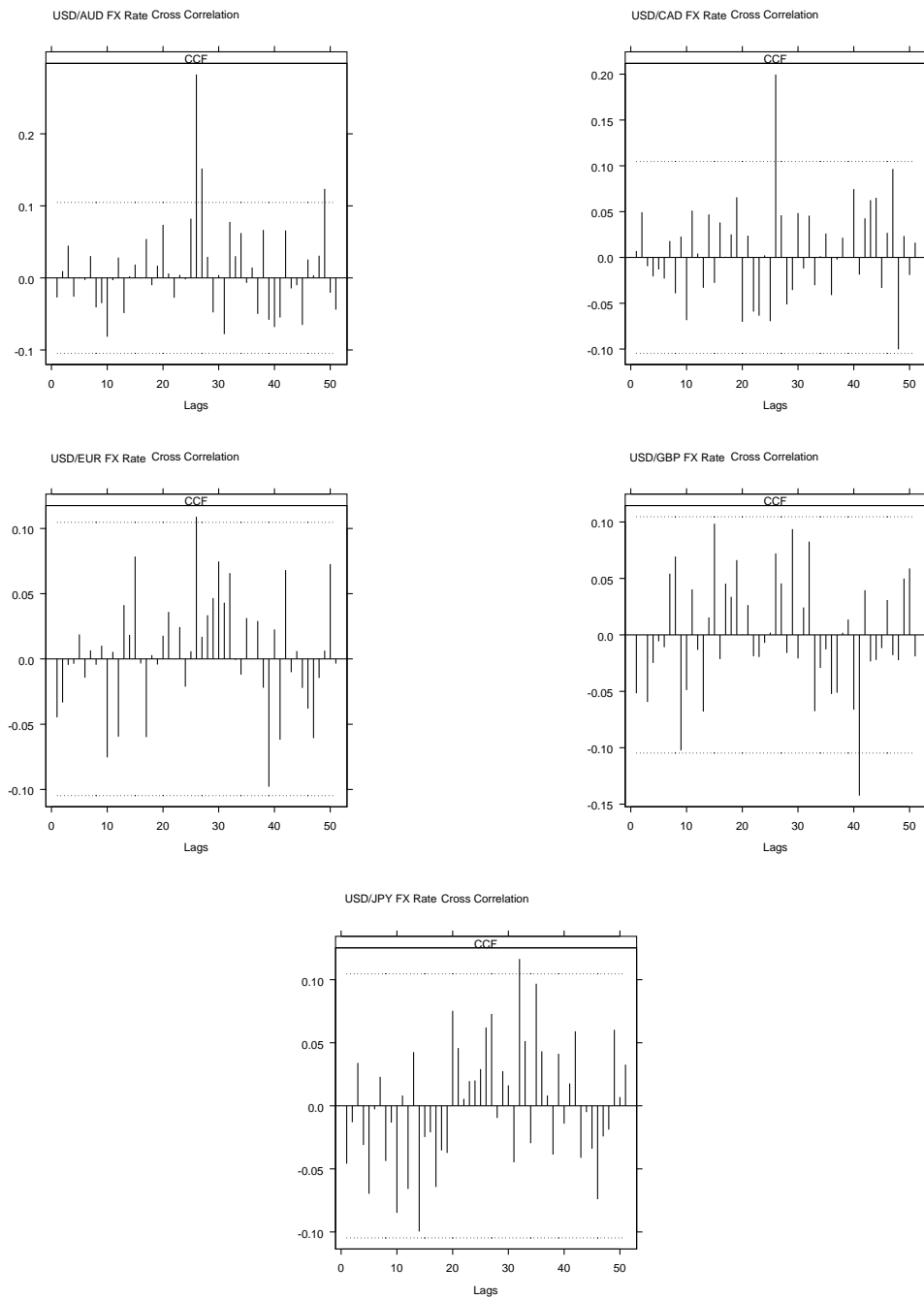


Figure 4.18: FX GARCH(1,1) Squared Residuals Cross Correlation with Residuals for 2004

Figure 4.18 better illustrates how the cross correlation between the FX GARCH(1,1) residuals and the squared residuals for each FX rate exhibit unique clustering over the time lags. Histograms for the standardised residuals were graphed in Figure 4.19.

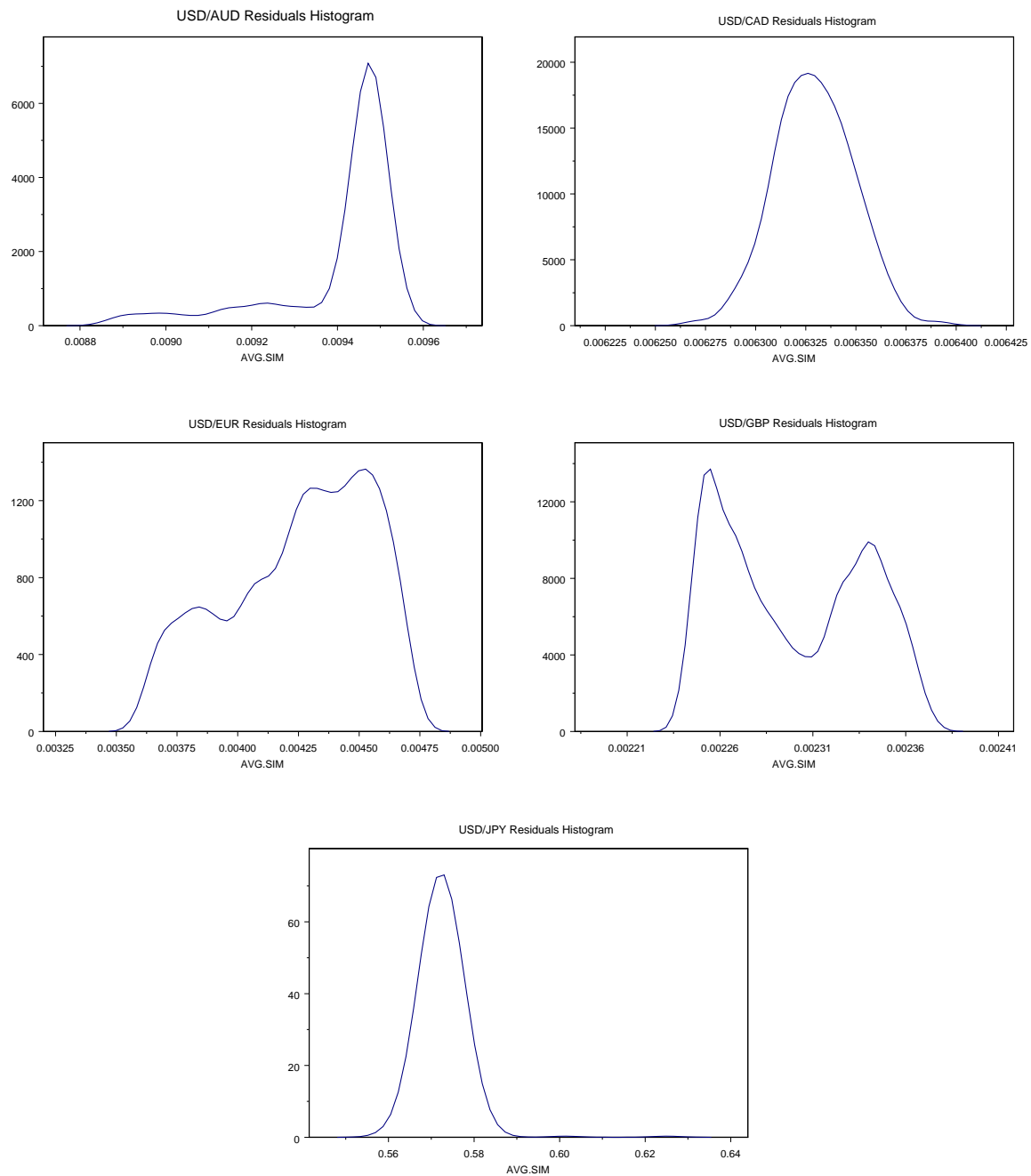
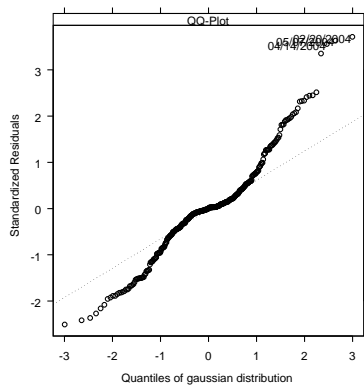


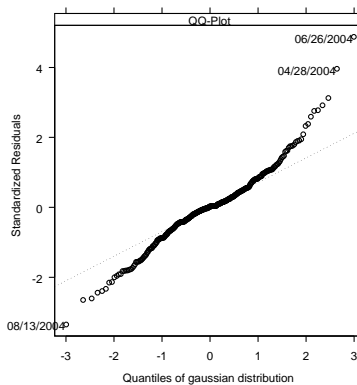
Figure 4.19: FX Histograms of Standardised GARCH(1,1) Residuals for 2004

Figure 4.19 shows that the standardised GARCH(1,1) residuals are unimodally distributed, except for USD/GBP which seems to be bimodally distributed. USD/EUR and USD/AUD had “fat tails” i.e. leptokurtosis. These standardised GARCH(1,1) residuals were then graphed in quantile plots in Figure 4.20.

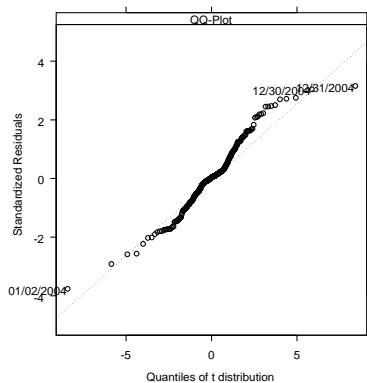
USD/AUD FX Rate QQ-Plot of Standardized Residuals



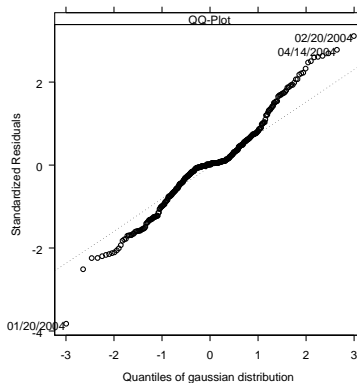
USD/CAD FX Rate QQ-Plot of Standardized Residuals



USD/EUR FX Rate QQ-Plot of Standardized Residuals



USD/GBP FX Rate QQ-Plot of Standardized Residuals



USD/JPY FX Rate QQ-Plot of Standardized Residuals

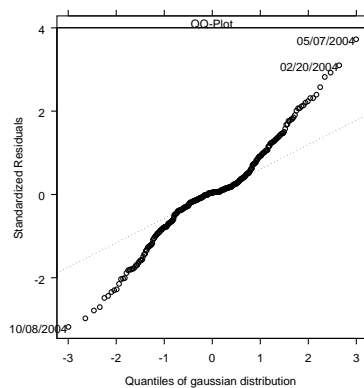


Figure 4.20: FX QQ-Plot of Standardised GARCH(1,1) Residuals for 2004

Figure 4.20 shows that the standardised GARCH(1,1) residuals fall mainly along the diagonal line showing that they are well described by the normal distribution. Now that we are confident with the chosen models, we list them in full detail with their parameters in Equations 4.2 to 4.6, with Equation 4.1 included as a reference.

$$\begin{aligned}
y_t &= c + y_{t-1} \\
\sigma_t^2 &= a_0 + \sum_{i=1}^p a_i \epsilon_{t-i}^2 + \sum_{j=1}^q b_j \sigma_{t-j}^2 \\
&= a_0 + a_1 \epsilon_{t-1}^2 + b_1 \sigma_{t-1}^2.
\end{aligned} \tag{4.1}$$

Note that since we are using GARCH(1,1), $p = 1 = q$.

USD/AUD

$$\begin{aligned}
y_t &= -0.00028050 + y_{t-1} \\
\sigma_t^2 &= 0.00000133 + 0.01715 \epsilon_{t-1}^2 + 0.9683 \sigma_{t-1}^2
\end{aligned} \tag{4.2}$$

USD/CAD

$$\begin{aligned}
y_t &= -0.00023507 + y_{t-1} \\
\sigma_t^2 &= 0.00003920 + 0.07073 \epsilon_{t-1}^2 - 0.0479 \sigma_{t-1}^2
\end{aligned} \tag{4.3}$$

USD/EUR

$$\begin{aligned}
y_t &= -0.00037730 + y_{t-1} \\
\sigma_t^2 &= 0.00000006 + 0.02012 \epsilon_{t-1}^2 + 0.9783 \sigma_{t-1}^2
\end{aligned} \tag{4.4}$$

USD/GBP

$$\begin{aligned}
y_t &= -0.00013860 + y_{t-1} \\
\sigma_t^2 &= 0.00000002 + 0.01417 \epsilon_{t-1}^2 + 0.9814 \sigma_{t-1}^2
\end{aligned} \tag{4.5}$$

USD/JPY

$$\begin{aligned}
y_t &= -0.03247000 + y_{t-1} \\
\sigma_t^2 &= 0.15177000 + 0.14566 \epsilon_{t-1}^2 + 0.3970 \sigma_{t-1}^2
\end{aligned} \tag{4.6}$$

To add the final level of detail regarding model fit, Tables 4.4 to 4.8 were included that detail the above parameters together with all the parameter estimate statistics.

Table 4.4: USD/AUD GARCH(1,1) Parameter Estimates for 2004

Estimated Coefficients	Value	Std. Error	t-Value	Pr(> t)
C	-0.0002805	0.0005252	-0.5342	0.29677
A	0.0000013	0.0000017	0.7810	0.21767
ARCH(1)	0.0171500	0.0130000	1.3186	0.09406
GARCH(1)	0.9683000	0.0281800	34.3651	0.00000
Information Criterion	AIC(4)	-2336.374	BIC(4)	-2320.774
Normality Test	Jarque-Bera 41.84	p-Value 0.00000000008	Shapiro-Wilk 0.9477	p-Value 0.0000000000000001
Ljung-Box Test for Standardized Residuals	Statistic 13.09	p-Value 0.3625	Chi² D.F. 12	
Ljung-Box Test for (Standardized Residuals)²	Statistic 29.84	p-Value 0.0030	Chi² D.F. 12	
Lagrange Multiplier Test	Lag 1 -0.1008	Lag 2 -1.6010	Lag 3 -2.2310	Lag 4 -0.0800
	Lag 5 0.8837	Lag 6 0.0057	Lag 7 1.2590	Lag 8 0.7548
	Lag 9 -1.8170	Lag 10 -1.6660	Lag 11 0.3764	Lag 12 0.1453
	C 0.09991			
	TR² 21.48	p-Value 0.04383	f-Stat 2.079	p-Value 0.0853

Table 4.5: USD/CAD GARCH(1,1) Parameter Estimates for 2004

Estimated Coefficients	Value	Std. Error	t-Value	Pr(> t)
C	-0.00023507	0.00033335	-0.7052	0.240575
A	0.00003924	0.00001595	2.4605	0.007171
ARCH(1)	0.07072875	0.03422492	2.0666	0.019743
GARCH(1)	-0.04786471	0.35762637	-0.1338	0.446802
Information Criterion	AIC(4)	-2649.051	BIC(4)	-2633.452
Normality Test	Jarque-Bera 97.23	p-Value 0	Shapiro-Wilk 0.9698	p-Value 0.0003322
Ljung-Box Test for Standardized Residuals	Statistic 24	p-Value 0.02037	Chi² D.F. 12	
Ljung-Box Test for (Standardized Residuals)²	Statistic 7.981	p-Value 0.7866	Chi² D.F. 12	
Lagrange Multiplier Test	Lag 1 0.1288	Lag 2 -1.362	Lag 3 -0.4821	Lag 4 -0.02432
	Lag 5 0.2271	Lag 6 -0.7185	Lag 7 0.2176	Lag 8 -0.4478
	Lag 9 -0.4709	Lag 10 -1.809	Lag 11 0.09417	Lag 12 0.06756
	C 0.7255			
	TR² 7.33	p-Value 0.835	f-Stat 0.6805	p-Value 0.8591

Table 4.6: USD/EUR GARCH(1,1) Parameter Estimates for 2004

Estimated Coefficients	Value	Std. Error	t-Value	Pr(> t)
C	-0.00037730	0.00019030	-1.983	0.02408
A	0.00000006	0.00000024	0.235	0.40716
ARCH(1)	0.02012000	0.01827000	1.102	0.13565
GARCH(1)	0.97830000	0.02077000	47.101	0.00000
Information Criterion	AIC(5)	-2928.597	BIC(5)	-2909.098
Normality Test	Jarque-Bera 38.75	p-Value 0.00000000385	Shapiro-Wilk 0.9586	p-Value 0.00000000449
Ljung-Box Test for Standardized Residuals	Statistic 15.45	p-Value 0.2177	Chi² D.F. 12	
Ljung-Box Test for (Standardized Residuals)²	Statistic 8.971	p-Value 0.7054	Chi² D.F. 12	
Lagrange Multiplier Test	Lag 1 -0.2028	Lag 2 0.07248	Lag 3 -1.416	Lag 4 -0.4309
	Lag 5 -0.2054	Lag 6 1.648	Lag 7 -0.6758	Lag 8 0.4172
	Lag 9 0.1315	Lag 10 -0.2626	Lag 11 -1.159	Lag 12 0.04616
	C 0.6664			
	TR² 7.845	p-Value 0.7972	f-Stat 0.7294	p-Value 0.8155

Table 4.7: USD/GBP GARCH(1,1) Parameter Estimates for 2004

Estimated Coefficients	Value	Std. Error	t-Value	Pr(> t)
C	-0.00013860	0.00014410	-0.9618	0.16839
A	0.00000002	0.00000007	0.3098	0.37846
ARCH(1)	0.01417000	0.01017000	1.3931	0.08223
GARCH(1)	0.98140000	0.01708000	57.4686	0.00000
Information Criterion	AIC(4)	-3229.259	BIC(4)	-3213.66
Normality Test	Jarque-Bera 8.224	p-Value 0.01638	Shapiro-Wilk 0.97	p-Value 0.0003973
Ljung-Box Test for Standardized Residuals	Statistic 8.707	p-Value 0.7278	Chi² D.F. 12	
Ljung-Box Test for (Standardized Residuals)²	Statistic 30.51	p-Value 0.00234	Chi² D.F. 12	
Lagrange Multiplier Test	Lag 1 -0.9377	Lag 2 0.7506	Lag 3 -2.021	Lag 4 -1.394
	Lag 5 0.4901	Lag 6 2.611	Lag 7 -0.2348	Lag 8 -1.473
	Lag 9 -2.659	Lag 10 -0.107	Lag 11 -0.9061	Lag 12 0.04919
	C 0.3873			
	TR² 26.22	p-Value 0.01	f-Stat 2.574	p-Value 0.03973

Table 4.8: USD/JPY GARCH(1,1) Parameter Estimates for 2004

Estimated Coefficients	Value	Std. Error	t-Value	Pr(> t)
C	-0.03247	0.02968	-1.094	0.1373752
A	0.15177	0.05470	2.775	0.0029063
ARCH(1)	0.14566	0.04281	3.403	0.0003712
GARCH(1)	0.39701	0.18274	2.173	0.0152330
Information Criterion	AIC(4)	625.9319	BIC(4)	641.5315
Normality Test	Jarque-Bera 26.53	p-Value 0.0000017	Shapiro-Wilk 0.9616	p-Value 0.0000001328
Ljung-Box Test for Standardized Residuals	Statistic 13.32	p-Value 0.3465	Chi² D.F. 12	
Ljung-Box Test for (Standardized Residuals)²	Statistic 24.53	p-Value 0.01719	Chi² D.F. 12	
Lagrange Multiplier Test	Lag 1 1.381	Lag 2 0.2303	Lag 3 -1.867	Lag 4 -0.9272
	Lag 5 2.47	Lag 6 1.835	Lag 7 0.8976	Lag 8 -0.5681
	Lag 9 -2.163	Lag 10 0.03663	Lag 11 -0.3819	Lag 12 0.2463
	C -0.9574			
	TR² 22.77	p-Value 0.02971	f-Stat 2.213	p-Value 0.06876

Tables 4.4 to 4.8 show that all models were correctly specified and are robust. This is evidenced, amongst other statistics by $(a_1 + b_1) < 1$, Shapiro-Wilk > 0.95 and the normality test having very small p-values (< 0.001), for all models. These tables also undertake Ljung-Box tests for standardised residuals, squared standardised residuals, Lagrange Multiplier tests for various lags. These tests further support the accuracy of the model parameter estimates.

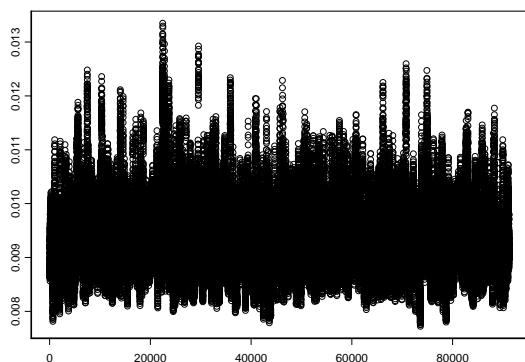
4.3 MC GARCH Simulations

By applying MC simulation to these estimated GARCH models, one can derive sufficient out-of-sample simulations to be able to generate the estimated distributions. These simulations can also be used to determine whether the resulting models reflect the behaviour of the original time series i.e. stress testing for model robustness.

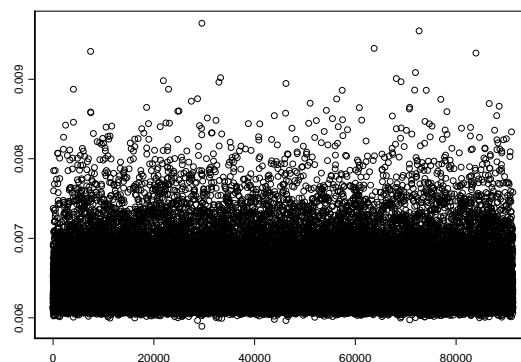
To verify this and to generate sufficient distributions to be netted, 250 MC simulations were generated at each point in time using random draws from a Gaussian ⁴ distribution. 250 MC simulations per each of the five FX rates was determined to be a suitable number of simulation runs without being either too few or too many. These are graphed using circle markers in Figure 4.21 and using line markers in Figure 4.22.

⁴Except for USD/EUR, in which the Student-t was used.

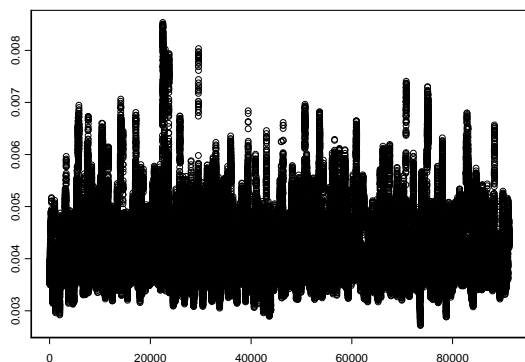
250 Simulations of USD/AUD GARCH(1,1) Model For 365 Days Out-Of-Sample



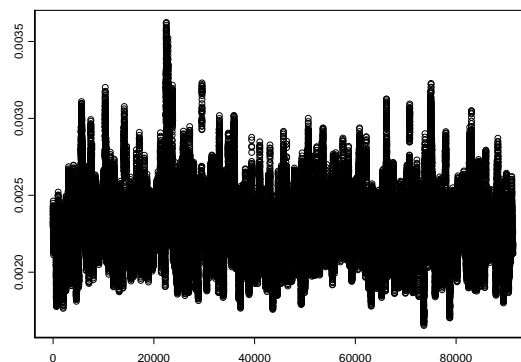
250 Simulations of USD/CAD GARCH(1,1) Model For 365 Days Out-Of-Sample



250 Simulations of USD/EUR GARCH(1,1) Model For 365 Days Out-Of-Sample



250 Simulations of USD/GBP GARCH(1,1) Model For 365 Days Out-Of-Sample



250 Simulations of USD/JPY GARCH(1,1) Model For 365 Days Out-Of-Sample

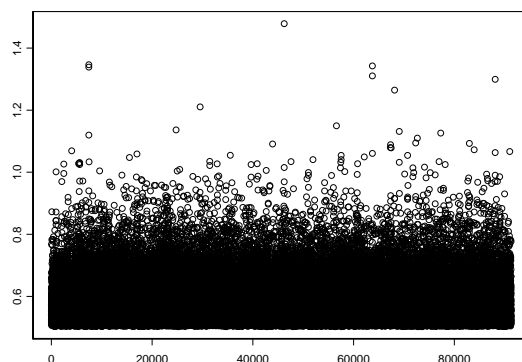
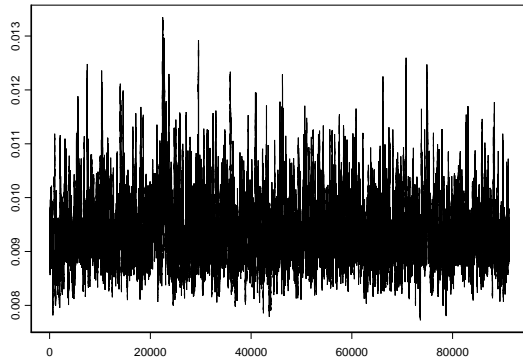
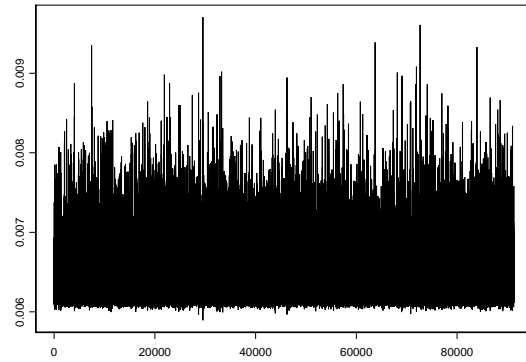


Figure 4.21: 250 MC Simulations of FX GARCH(1,1) Models for 2005 Using Circle Markers

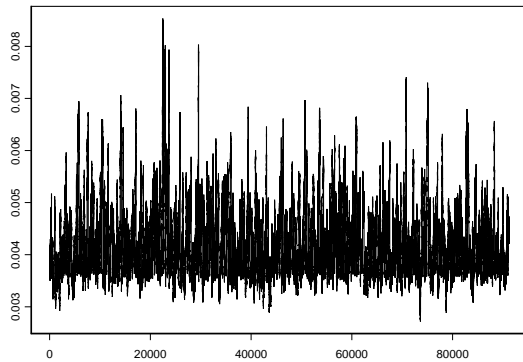
250 Simulations of USD/AUD GARCH(1,1) Model For 365 Days Out-Of-Sample



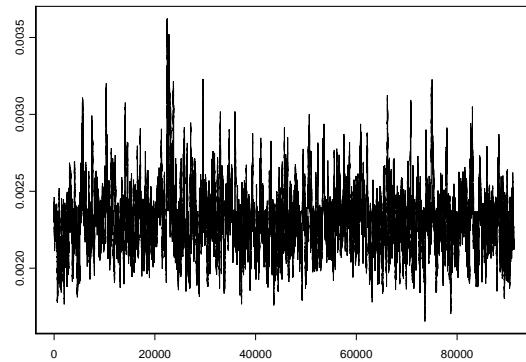
250 Simulations of USD/CAD GARCH(1,1) Model For 365 Days Out-Of-Sample



250 Simulations of USD/EUR GARCH(1,1) Model For 365 Days Out-Of-Sample



250 Simulations of USD/GBP GARCH(1,1) Model For 365 Days Out-Of-Sample



250 Simulations of USD/JPY GARCH(1,1) Model For 365 Days Out-Of-Sample

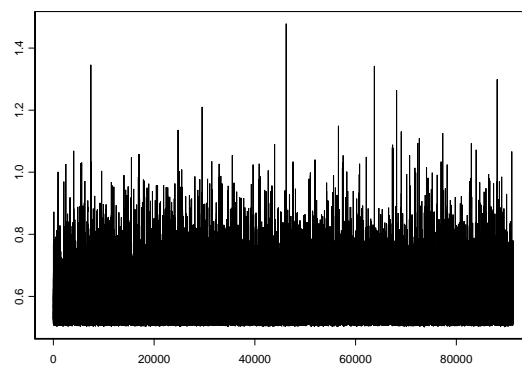


Figure 4.22: 250 MC Simulations of FX GARCH(1,1) Models for 2005 Using Line Markers

Figure 4.21 and Figure 4.21 show that the MC simulations do not exhibit any anomalies that would suggest that they were not representative of the original FX rate volatility. The circle markers of Figure 4.21 have the advantage over the Figure 4.22 version as they make it easier to distinguish the contribution of each simulation point by having less overlap. The line markers of Figure 4.22 have the advantage over the 4.21 version as they are more narrower and it is more obvious how they coincide with the volatility of Figure 4.11. This is most apparent with USD/CAD and USD/JPY rates as they are more significantly bound from below, and so the other three FX rates have less skew. Furthermore, all simulations emanate or evolve from a certain point unique to each FX rate. These points are the last point of the daily differences from 2004 (i.e. 31/12/2004). Figure 4.22 was then clarified by only showing the *first* 10 of the 250 simulations, together with the mean in bold red to produce Figure 4.23.

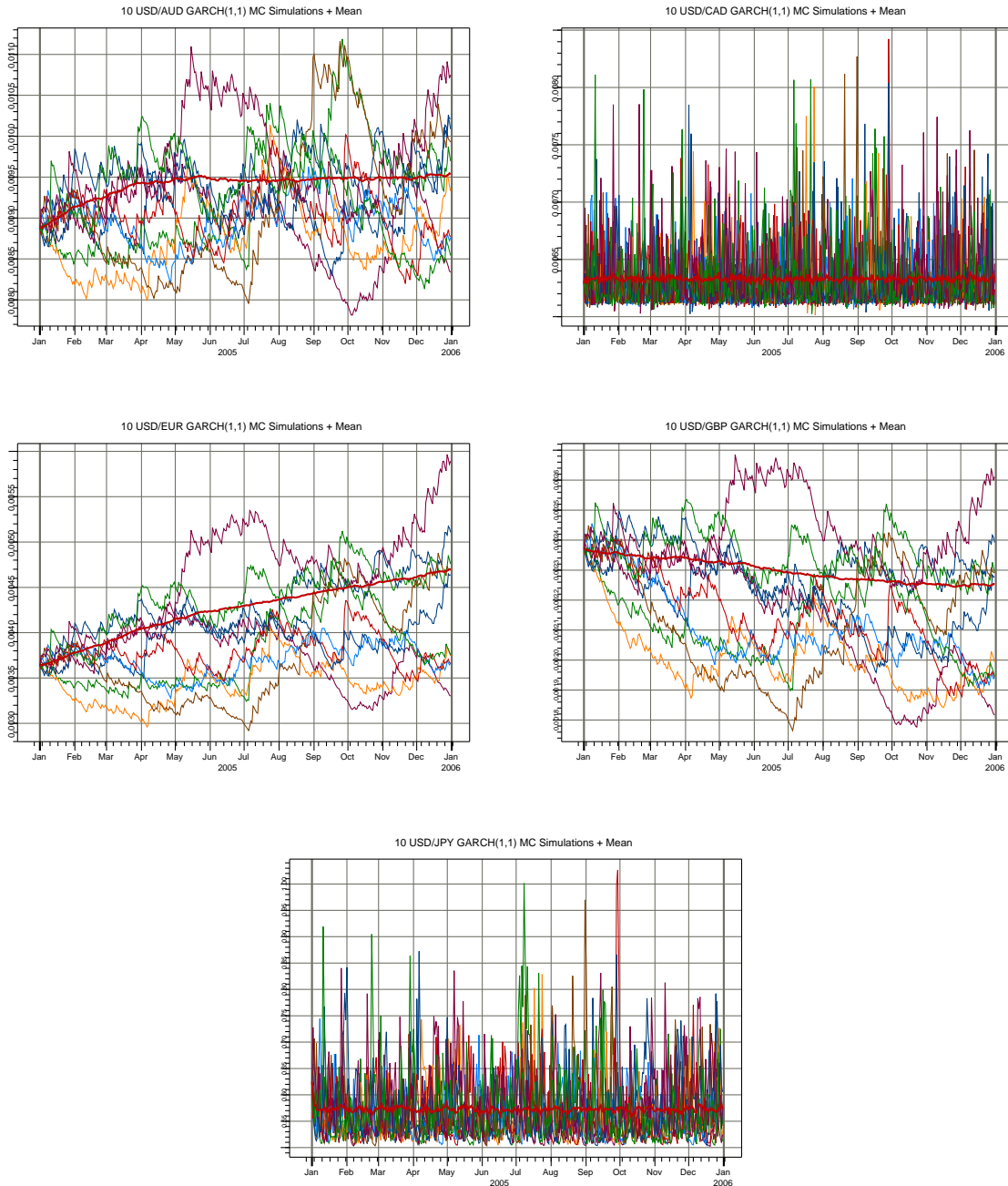


Figure 4.23: First 10 MC Simulations of FX GARCH(1,1) Models + Mean (In Bold Red) for 2005

The simulations in Figure 4.23 are well behaved and do not exhibit any anomalies, hence reinforcing our confidence in the MC approach adopted. Figure 4.23 was reproduced for the *last* 10 of the 250 simulations and is shown in Figure 4.24 to help validate whether Figure 4.23 was representative of the true distributions.

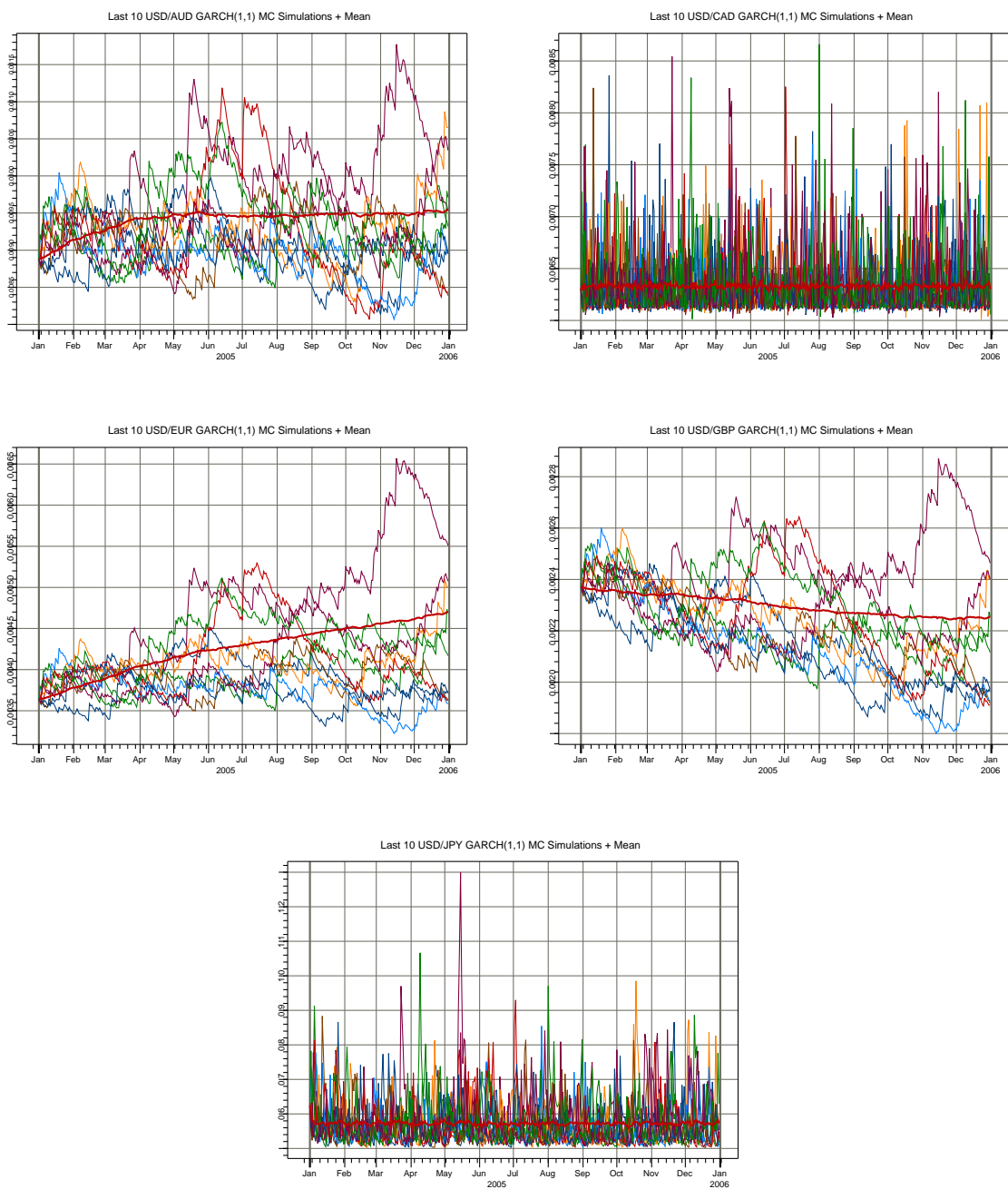


Figure 4.24: Last 10 MC Simulations of FX GARCH(1,1) Models + Mean (In Bold Red) for 2005

Figure 4.24 suggests that there is nothing obvious to argue that it behaves differently to the simulations of Figure 4.23. To obtain a more rigorous measure of the mean and variance of these simulations for more than 250 simulation runs, Figure 4.25 was graphed, showing the predicted conditional standard deviation.

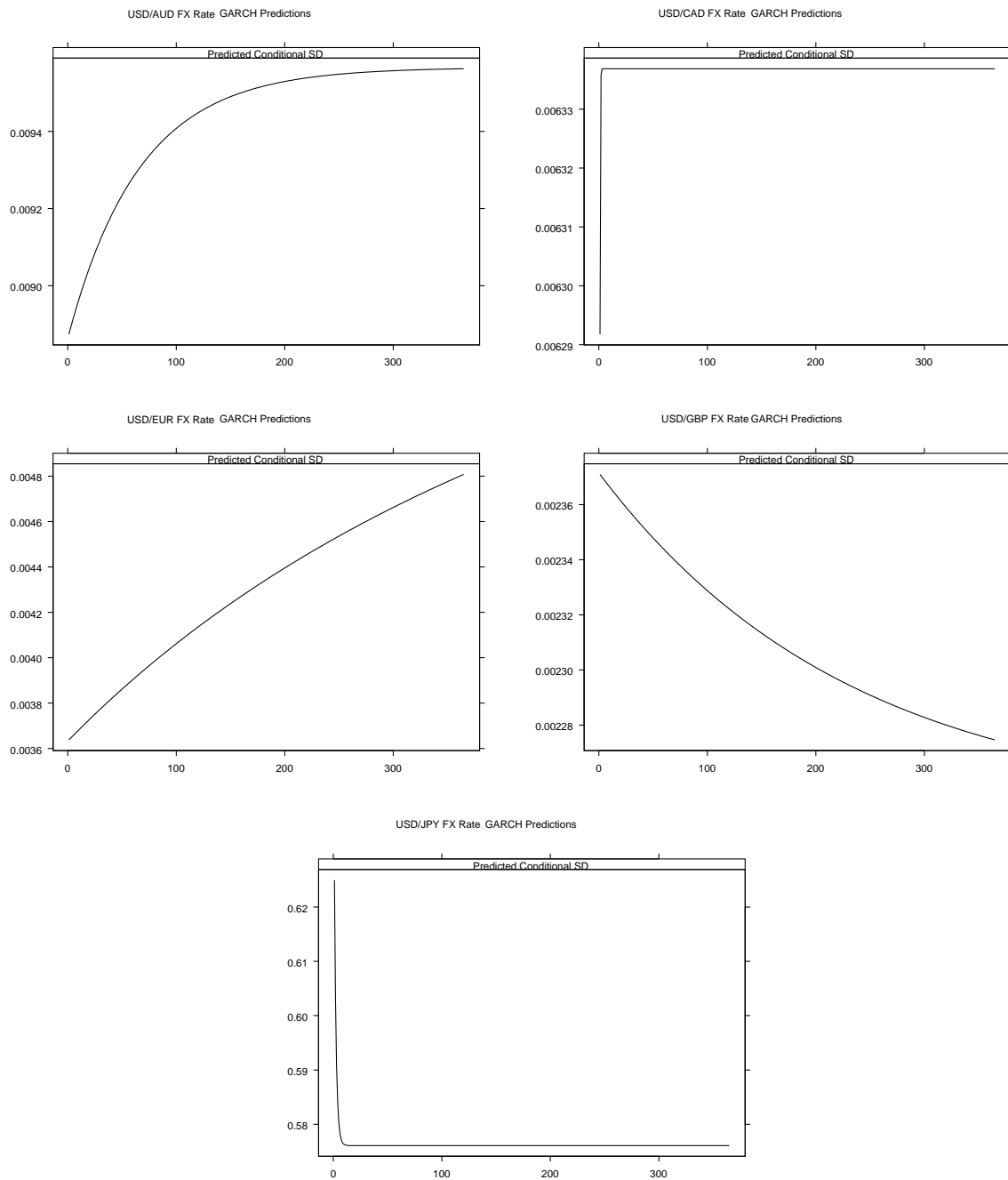


Figure 4.25: FX GARCH(1,1) Predicted Conditional Standard Deviation for 2005

Figure 4.25 shows the conversion rate and model behaviour for each of the FX GARCH(1,1) models. The USD/AUD, USD/CAD and USD/EUR GARCH models all increase over time, whilst USD/GBP and USD/JPY decrease over time. Some of these models converge at a much faster rate than the others, namely the USD/CAD and the USD/JPY models. However, some may think that the decline of the USD/GBP and USD/JPY models may imply that these models are not performing as well as the others. This is not the case, and so the simulated confidence interval of FX volatility was graphed in Figure 4.26 to provide a further measure of the simulations' variation.

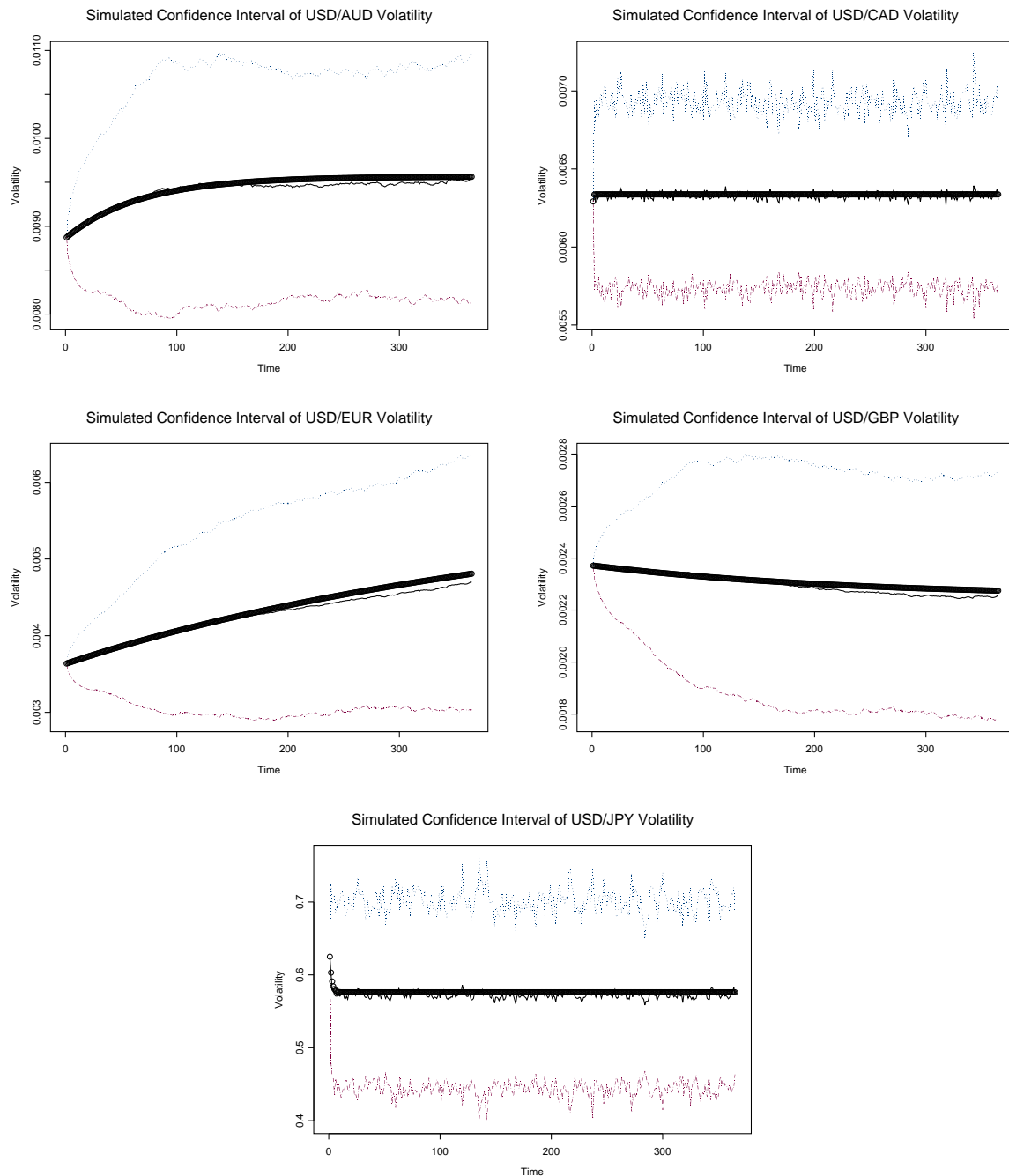


Figure 4.26: FX GARCH(1,1) Volatility MC Simulated Confidence Interval for 2005

Figure 4.26 displays the nature of the GARCH(1,1) models' predictions by showing the minima, maxima and mean for the simulated volatility. Figure 4.26 more clearly shows that all five models converge, especially by the 150th day out-of-sample.

4.4 Application of Netting Model

We will examine the netting model applied to our 250 FX GARCH(1,1) volatility MC simulations. Table 4.9 shows the first 10 daily data points for the first simulation run of our five FX rates. This data formed one of the curves that was used for Figure 4.23.

Table 4.9: First 10 MC Simulations of FX GARCH(1,1) Models for 1st of 250 Simulation Runs for 2005 in Business Notation

Simulation Run	Day Point	USD /AUD	USD /CAD	USD /EUR	USD /GBP	USD /JPY
1	01-Jan-05	0.008874078	0.006291776	0.003637556	0.002370946	0.624975500
1	02-Jan-05	0.008811324	0.006120256	0.003607211	0.002354252	0.556023500
1	03-Jan-05	0.008771435	0.006188524	0.003587522	0.002342459	0.537590700
1	04-Jan-05	0.008711624	0.006126340	0.003558033	0.002326182	0.518152400
1	05-Jan-05	0.008656230	0.006137661	0.003530341	0.002310740	0.511602400
1	06-Jan-05	0.008737517	0.006521695	0.003567791	0.002325384	0.573256500
1	07-Jan-05	0.008710728	0.006215328	0.003554002	0.002316384	0.552231500
1	08-Jan-05	0.008654331	0.006130887	0.003525867	0.002300822	0.525426300
1	09-Jan-05	0.008649826	0.006279872	0.003522271	0.002296545	0.539993600
1	10-Jan-05	0.008871216	0.006931615	0.003626441	0.002342032	0.656065600

Using the netting notation of **Chapter 3**, Table 4.9 can be expressed in more generalised terms as Table 4.10.

Table 4.10: First 10 MC Simulations of FX GARCH(1,1) Models for 1st of 250 Simulation Runs for 2005 in Netting Notation

Simulation Run	t	$e_{1,1}(t)$	$e_{2,2}(t)$	$e_{3,3}(t)$	$e_{4,4}(t)$	$e_{5,5}(t)$
1	1	0.008874078	0.006291776	0.003637556	0.002370946	0.624975500
1	2	0.008811324	0.006120256	0.003607211	0.002354252	0.556023500
1	3	0.008771435	0.006188524	0.003587522	0.002342459	0.537590700
1	4	0.008711624	0.006126340	0.003558033	0.002326182	0.518152400
1	5	0.008656230	0.006137661	0.003530341	0.002310740	0.511602400
1	6	0.008737517	0.006521695	0.003567791	0.002325384	0.573256500
1	7	0.008710728	0.006215328	0.003554002	0.002316384	0.552231500
1	8	0.008654331	0.006130887	0.003525867	0.002300822	0.525426300
1	9	0.008649826	0.006279872	0.003522271	0.002296545	0.539993600
1	10	0.008871216	0.006931615	0.003626441	0.002342032	0.656065600

Table 4.10 lists the first simulation run in a more similar mathematical format. If we look at

the first simulation run, for day 1 (i.e. for $t=1$), this will populate only the diagonals of the netting matrix, i.e. $e_{i,j}(1)$ when $i = j$, but not when $i \neq j$, as shown in Equation 4.7.

$$E = \begin{bmatrix} e_{1,1}(1) & e_{1,2}(1) & e_{1,3}(1) & e_{1,4}(1) & e_{1,5}(1) \\ e_{2,1}(1) & e_{2,2}(1) & e_{2,3}(1) & e_{2,4}(1) & e_{2,5}(1) \\ e_{3,1}(1) & e_{3,2}(1) & e_{3,3}(1) & e_{3,4}(1) & e_{3,5}(1) \\ e_{4,1}(1) & e_{4,2}(1) & e_{4,3}(1) & e_{4,4}(1) & e_{4,5}(1) \\ e_{5,1}(1) & e_{5,2}(1) & e_{5,3}(1) & e_{5,4}(1) & e_{5,5}(1) \end{bmatrix}. \quad (4.7)$$

So far, we know the values of $e_{1,1}(1)$, $e_{1,2}(1)$, $e_{1,3}(1)$, $e_{1,4}(1)$, $e_{1,5}(1)$ from Table 4.10, just its first row. However, we notice from Equation 3.7 that $e_{i,j}(1) = 0$ where $i = j$, as a bank does not have an exposure with itself. We also attribute each bank's simulated MC exposure to the other four counterparty banks equally. Hence, Equation 4.7 can be expressed as Equation 4.8.

$$E = \left[\begin{array}{cc|ccc} 0 & e_{1,1}(1)/4 & e_{1,1}(1)/4 & e_{1,1}(1)/4 & e_{1,1}(1)/4 \\ e_{2,2}(1)/4 & 0 & e_{2,2}(1)/4 & e_{2,2}(1)/4 & e_{2,2}(1)/4 \\ e_{3,3}(1)/4 & e_{3,3}(1)/4 & 0 & e_{3,3}(1)/4 & e_{3,3}(1)/4 \\ e_{4,4}(1)/4 & e_{4,4}(1)/4 & e_{4,4}(1)/4 & 0 & e_{4,4}(1)/4 \\ e_{5,5}(1)/4 & e_{5,5}(1)/4 & e_{5,5}(1)/4 & e_{5,5}(1)/4 & 0 \end{array} \right]. \quad (4.8)$$

Again, notice that for $i \neq j$, $e_{i,j}(1)$ in Equation 4.7 maps to $e_{i,i}(1)/4$ in Equation 4.8. This is because this thesis has taken the approach of each bank being equally exposed to the other banks. Since we are considering 5 banks in total, each bank is thus exposed equally amongst the remaining 4 banks. Although such an approach may seem to some as somewhat naive, it is actually deliberate as there is practically infinitely many arrangements between n banks. For this extreme, as n increases, the number of arrangements grows exponentially which is not feasible. On the other extreme, just having only one arrangement is not feasible as it would produce non-generalised conclusions. This then justifies our approach as the most natural arrangement that avoids the pitfalls of the above two possible extremes.

Notice also the vertical line that separates columns 1 and 2 from the rest of the matrix. This will become more apparent below, but it is present essentially to help simplify the notation and to prevent the matrix from taking up more than a page of A4 paper. This lead to the nota-

tion $A_1(1)...A_{12}(1)$ and $B_1(1)...B_{12}(1)$ to reduce the size of Equation 4.9 below. Consequently, Equation 4.8 becomes Equation 4.9 when we substitute the values from Table 4.10 further below.

$$\begin{aligned}
N(E) &= N \left(\begin{bmatrix} 0 & e_{1,1}(1)/4 & e_{1,1}(1)/4 & e_{1,1}(1)/4 & e_{1,1}(1)/4 \\ e_{2,2}(1)/4 & 0 & e_{2,2}(1)/4 & e_{2,2}(1)/4 & e_{2,2}(1)/4 \\ e_{3,3}(1)/4 & e_{3,3}(1)/4 & 0 & e_{3,3}(1)/4 & e_{3,3}(1)/4 \\ e_{4,4}(1)/4 & e_{4,4}(1)/4 & e_{4,4}(1)/4 & 0 & e_{4,4}(1)/4 \\ e_{5,5}(1)/4 & e_{5,5}(1)/4 & e_{5,5}(1)/4 & e_{5,5}(1)/4 & 0 \end{bmatrix} \right) \\
&= \begin{bmatrix} 0 & \max\{e_{1,1}(1)/4 - e_{2,2}(1)/4, 0\} & A_1(1) & A_2(1) & A_3(1) \\ \max\{e_{2,2}(1)/4 - e_{1,1}(1)/4, 0\} & 0 & A_4(1) & A_5(1) & A_6(1) \\ \max\{e_{3,3}(1)/4 - e_{1,1}(1)/4, 0\} & \max\{e_{3,3}(1)/4 - e_{2,2}(1)/4, 0\} & 0 & A_7(1) & A_8(1) \\ \max\{e_{4,4}(1)/4 - e_{1,1}(1)/4, 0\} & \max\{e_{4,4}(1)/4 - e_{2,2}(1)/4, 0\} & A_9(1) & 0 & A_{10}(1) \\ \max\{e_{5,5}(1)/4 - e_{1,1}(1)/4, 0\} & \max\{e_{5,5}(1)/4 - e_{2,2}(1)/4, 0\} & A_{11}(1) & A_{12}(1) & 0 \end{bmatrix} \\
&= \begin{bmatrix} 0 & \max\{0.002218520 - 0.001572944, 0\} & B_1(1) & B_2(1) & B_3(1) \\ \max\{0.001572944 - 0.002218520, 0\} & 0 & B_4(1) & B_5(1) & B_6(1) \\ \max\{0.000909389 - 0.002218520, 0\} & \max\{0.000909389 - 0.001572944, 0\} & 0 & B_7(1) & B_8(1) \\ \max\{0.000592737 - 0.002218520, 0\} & \max\{0.000592737 - 0.001572944, 0\} & B_9(1) & 0 & B_{10}(1) \\ \max\{0.156243875 - 0.002218520, 0\} & \max\{0.156243875 - 0.001572944, 0\} & B_{11}(1) & B_{12}(1) & 0 \end{bmatrix} \\
&= \begin{bmatrix} 0 & 0.000645576 & 0.001309131 & 0.001625783 & 0 \\ 0 & 0 & 0.000663555 & 0.000980207 & 0 \\ 0 & 0 & 0 & 0.000316652 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0.154025355 & 0.154670931 & 0.155334486 & 0.155651138 & 0 \end{bmatrix}.
\end{aligned}
\tag{4.9}$$

From Equation 4.9, we can deduce the values for $A_1(1) \dots A_{12}(1)$ as,

$$\begin{aligned}
A_1(1) &= \max\{e_{1,1}(1)/4 - e_{3,3}(1)/4, 0\} \\
A_2(1) &= \max\{e_{1,1}(1)/4 - e_{4,4}(1)/4, 0\} \\
A_3(1) &= \max\{e_{1,1}(1)/4 - e_{5,5}(1)/4, 0\} \\
A_4(1) &= \max\{e_{2,2}(1)/4 - e_{3,3}(1)/4, 0\} \\
A_5(1) &= \max\{e_{2,2}(1)/4 - e_{4,4}(1)/4, 0\} \\
A_6(1) &= \max\{e_{2,2}(1)/4 - e_{5,5}(1)/4, 0\} \\
A_7(1) &= \max\{e_{3,3}(1)/4 - e_{4,4}(1)/4, 0\} \\
A_8(1) &= \max\{e_{3,3}(1)/4 - e_{5,5}(1)/4, 0\} \\
A_9(1) &= \max\{e_{4,4}(1)/4 - e_{3,3}(1)/4, 0\} \\
A_{10}(1) &= \max\{e_{4,4}(1)/4 - e_{5,5}(1)/4, 0\} \\
A_{11}(1) &= \max\{e_{5,5}(1)/4 - e_{3,3}(1)/4, 0\} \\
A_{12}(1) &= \max\{e_{5,5}(1)/4 - e_{4,4}(1)/4, 0\}.
\end{aligned}$$

and through Table B.10 we can deduce the values for $B_1(1) \dots B_{12}(1)$ as,

$$\begin{aligned}
B_1(1) &= \max\{0.002218520 - 0.000909389, 0\} \\
&= 0.001309131. \\
B_2(1) &= \max\{0.002218520 - 0.000592737, 0\} \\
&= 0.001625783. \\
B_3(1) &= \max\{0.002218520 - 0.156243875, 0\} \\
&= 0. \\
B_4(1) &= \max\{0.001572944 - 0.000909389, 0\} \\
&= 0.000663555. \\
B_5(1) &= \max\{0.001572944 - 0.000592737, 0\} \\
&= 0.000980207. \\
B_6(1) &= \max\{0.001572944 - 0.156243875, 0\} \\
&= 0. \\
B_7(1) &= \max\{0.000909389 - 0.000592737, 0\} \\
&= 0. \\
B_8(1) &= \max\{0.000909389 - 0.156243875, 0\} \\
&= 0.000316652. \\
B_9(1) &= \max\{0.000592737 - 0.000909389, 0\} \\
&= 0. \\
B_{10}(1) &= \max\{0.000592737 - 0.156243875, 0\} \\
&= 0.155334486. \\
B_{11}(1) &= \max\{0.156243875 - 0.000909389, 0\} \\
&= 0.155651138. \\
B_{12}(1) &= \max\{0.156243875 - 0.000592737, 0\} \\
&= 0.
\end{aligned}$$

We now can apply the formula for a bank's total exposure as seen in **Appendix - Banks Total Exposure** to arrive at Equation 4.10 below.

$$\begin{aligned}
T(N(E)) &= T \left(\left[\begin{array}{cc|cc} 0 & 0.000645576 & 0.001309131 & 0.001625783 & 0 \\ 0 & 0 & 0.000663555 & 0.000980207 & 0 \\ 0 & 0 & 0 & 0.000316652 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0.154025355 & 0.154670931 & 0.155334486 & 0.155651138 & 0 \end{array} \right] \right) \\
&= \left[\begin{array}{cc|cc} 0 & 0.000645576 & 0.001309131 & 0.001625783 & 0 \\ 0 & 0 & 0.000663555 & 0.000980207 & 0 \\ 0 & 0 & 0 & 0.000316652 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0.154025355 & 0.154670931 & 0.155334486 & 0.155651138 & 0 \end{array} \right] \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \\ 1 \end{bmatrix} \\
&= \left[\begin{array}{l} 0 + 0.000645576 + 0.001309131 + 0.001625783 + 0 \\ 0 + 0 + 0.000663555 + 0.000980207 + 0 \\ 0 + 0 + 0 + 0.000316652 + 0 \\ 0 + 0 + 0 + 0 + 0 \\ 0.154025355 + 0.154670931 + 0.155334486 + 0.155651138 + 0 \end{array} \right] \\
&= \left[\begin{array}{l} 0.00358049 \\ 0.00164376 \\ 0.00031665 \\ 0 \\ 0.61968191 \end{array} \right] \cdot \quad (4.10)
\end{aligned}$$

From Equation 4.10, we see that this is just for when $t = 1$, i.e. the first simulated day of a total 365. When $t = 2$, we can calculate Equation 4.10 again, and so on until $t = 365$ (i.e. 31/12/2005). The result of applying this netting methodology for the MC GARCH(1,1) simulated 2005 data is shown in Figure 4.27.

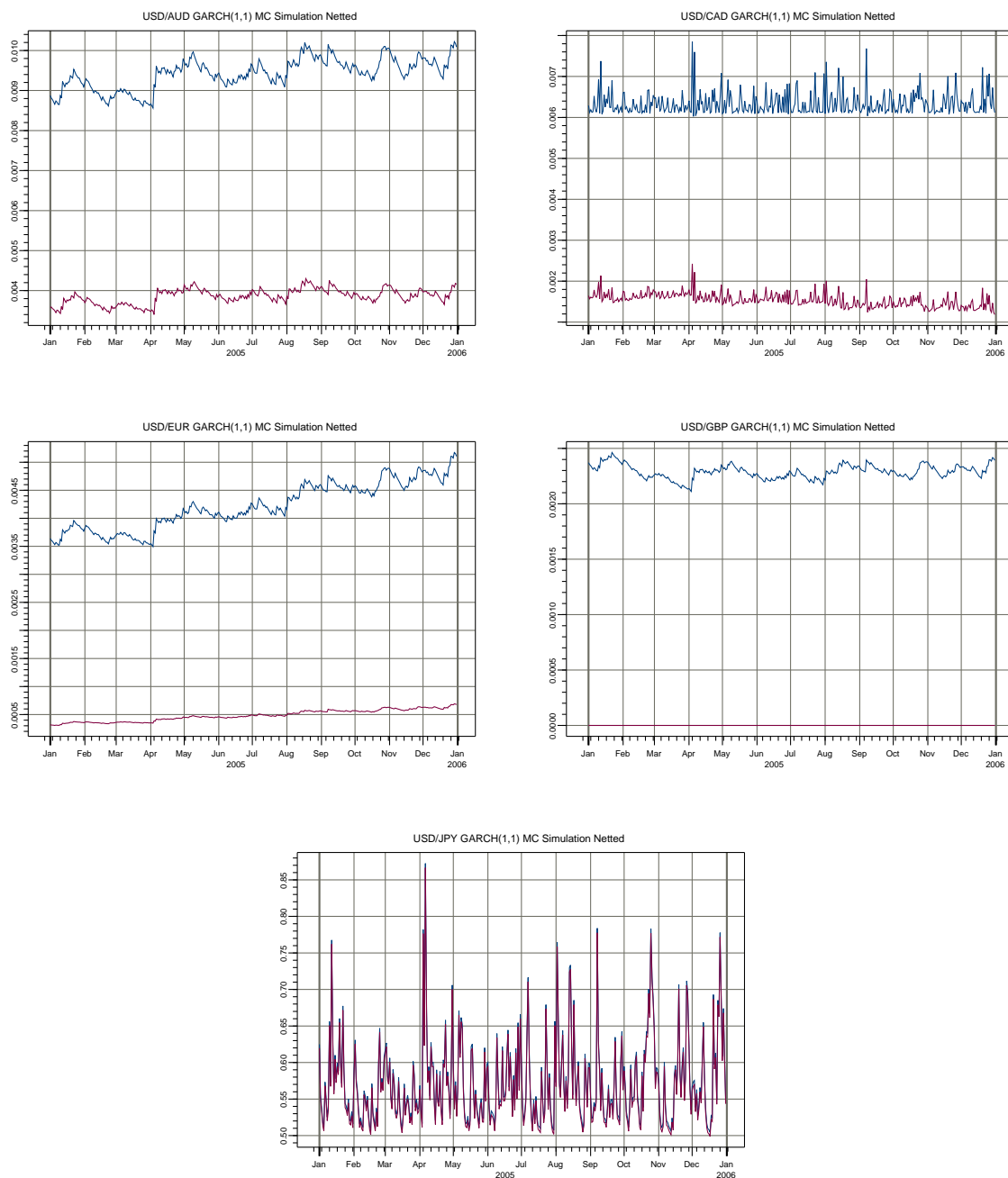


Figure 4.27: First MC Simulation of FX GARCH(1,1) Models Netted for 2005 for 5 Banks

Figure 4.27 shows that the FX rate exposures for all 5 hypothetical international banks both with and without netting on the same graph. Notice that the exposures are significantly reduced due to netting, except for USD/JPY. USD/JPY has actually had very little change, and USD/GBP has had an exposure of zero so it is difficult to notice. Such a result is due to the magnitudes of the FX rate exposures and their multivariate relation to the other FX rate exposures. Notice also that netting is not a simple linear translation or shift function (i.e. $N[e(t)] \neq e(t) - h$ for some netting function N applied to some exposure e over time t cannot be expressed in terms of some shift constant h). This observation is most obvious in the USD/EUR exposure.

To determine whether the result for USD/JPY was just an anomaly, an alternative arrangement was established between the other 4 FX currencies, thus creating a system of 4 rates rather than 5. This is shown in Figure 4.28.



Figure 4.28: First MC Simulation of FX GARCH(1,1) Models Netted for 2005 for 4 Banks

Surprisingly, Figure 4.28 shows that the USD/GBP still has an exposure of zero after netting.

This suggests that depending on the magnitudes of other FX exposures, one bank may always be netted to zero. To see whether these results for 5 and 4 banks was present in only the 1st simulation run or over all 250 simulation runs, the resulting histograms together with their density curves are plotted in Figure 4.29 and Figure 4.30.

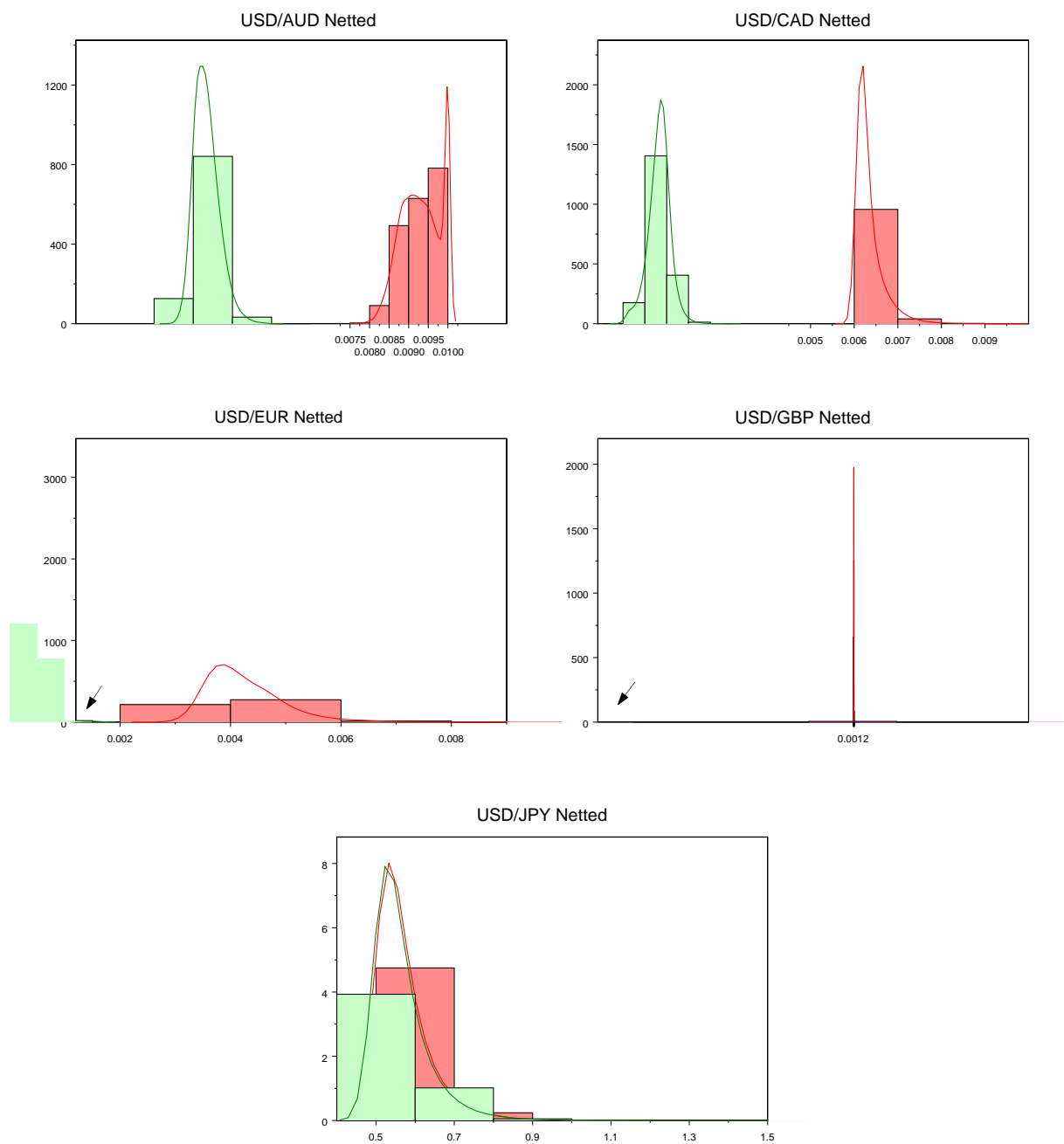


Figure 4.29: MC Simulation Distribution of 250 FX GARCH(1,1) Models Netted for 2005 for 5 Banks

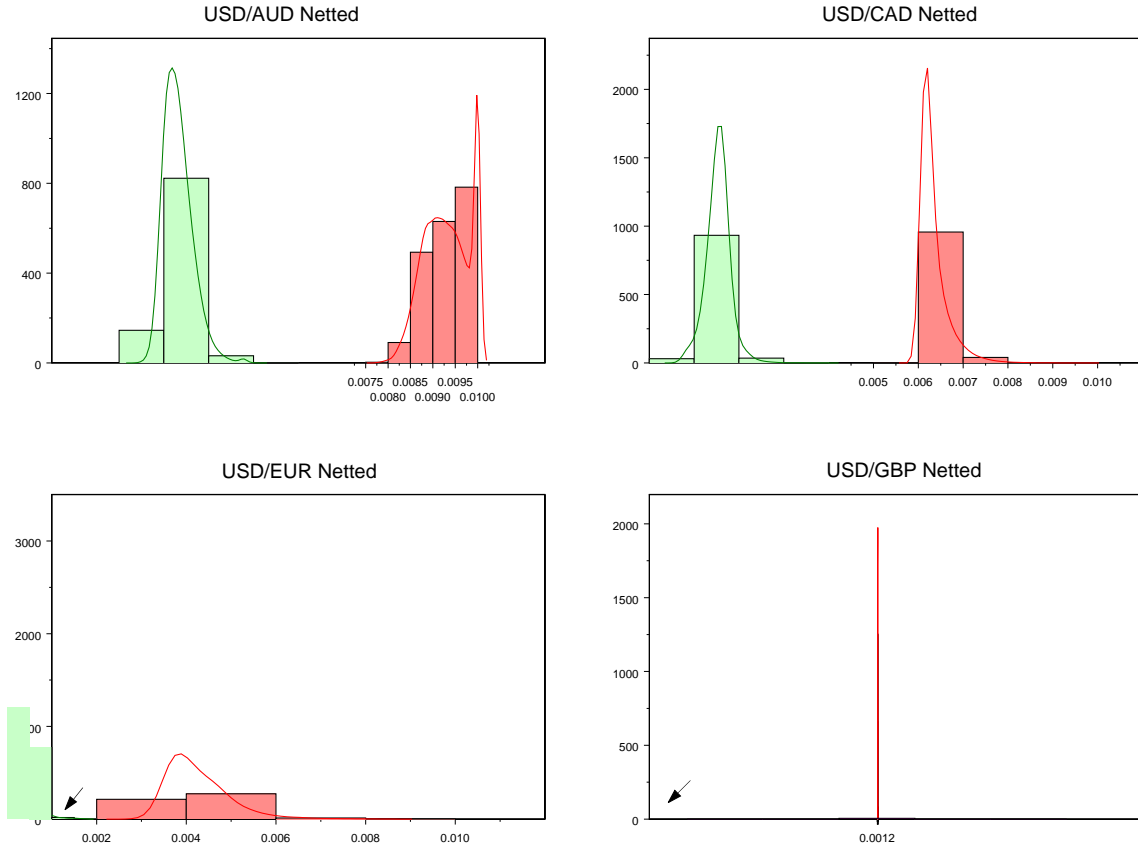


Figure 4.30: MC Simulation Distribution of 250 FX GARCH(1,1) Models Netted for 2005 for 4 Banks

From Figures 4.29 and 4.30, we see that from all distributions, netting has reduced their mean and variance. We also see that the results for 1 simulation run above were consistent with 250 simulation runs. For the cases where there is very little change or a significant reduction to zero, such a pattern is difficult to notice or visually deduce, hence the use of arrows to point to the resulting distribution. To determine whether these differences were statistically significant, a Two Sample t-Test was carried out for each of the five FX rates, as shown in Table 4.11.

Table 4.11: Two Sample t-Test for 5 Banks Both With and Without Netting

Rate	μ	Non-Netted	σ	Non-Netted	μ	Netted	σ	Netted	t	D.F.	p-Value	95% C.I. Start	95% C.I. End
USD/AUD	0.009323185	0.0004978054	0.0004978054	0.003833200	0.003833200	0.0003174643	0.0003174643	0.0003174643	2808.845	182498	0	0.005486154	0.005493816
USD/CAD	0.006329099	0.0002990292	0.0002990292	0.001528931	0.001528931	0.0002451115	0.0002451115	0.0002451115	3750.204	182498	0	0.004797659	0.004802676
USD/EUR	0.004253520	0.0006956116	0.0006956116	0.000491142	0.000491142	0.0001466877	0.0001466877	0.0001466877	1598.690	182498	0	0.003757765	0.003766991
USD/GBP	0.002298858	0.0002153728	0.0002153728	0.000000000	0.000000000	0.0000000000	0.0000000000	0.0000000000	3224.316	182498	0	0.002297460	0.002300255
USD/JPY	0.567532600	0.0647314300	0.0647314300	0.561965000	0.561965000	0.0645234300	0.0645234300	0.0645234300	18.401	182498	0	0.004974545	0.006160578

Table 4.11 shows that all currencies except USD/JPY have high degrees of freedom yet very narrow confidence intervals at the 95% significance level. Together with the other statistics, this suggests that that the impact of netting is a statistically significant process, at least for the first four currencies. However, the t-Tests displayed in Table 4.11 assumed that the samples or in this case the distributions are Gaussian. Our netted data is not Gaussian, and so a more generic Two Sample Wilcoxon Rank-Sum Test was carried out for each of the five FX rates, as shown in Table 4.12.

Table 4.12: Two Sample Wilcoxon Rank-Sum Test for 5 Banks Both With and Without Netting

Rate	μ	Non-Netted	σ	Non-Netted	μ	Netted	σ	Netted	Z	p-Value
USD/AUD	0.009323185	0.0004978054	0.003833200	0.003174643	370.0767	0				
USD/CAD	0.006329099	0.0002990292	0.001528931	0.0002451115	369.9652	0				
USD/EUR	0.004253520	0.0006956116	0.000491142	0.0001466877	369.9652	0				
USD/GBP	0.002298858	0.0002153728	0.000000000	0.000000000	395.5094	0				
USD/JPY	0.567532600	0.0647314300	0.561965000	0.0645234300	28.9059	0				

The Two Sample Wilcoxon Rank-Sum Test of Table 4.12 is further evidence that except for USD/JPY, netting has produced a statistically significant impact on Bank's FX derivatives and their corresponding portfolio.

In summary, this chapter has analysed the FX rate time series data and determined its characteristics prior to modelling. These five datasets were then used to estimate five GARCH models using the normal distribution. One model required a departure from the normal distribution -namely the Student-t distribution. The model parameters were examined for suitability by performing a number of goodness-of-fit tests. The models were then simulated to produce the non-netted FX time series data. Then the netting model was applied to this data to produce the netted FX time series data. The results were compared to estimate the impact due to close-out netting.

In preparation for the concluding chapter, it is useful to review the results through one of three following possible risks within the findings;

1. If coefficients are not significant, they should be eliminated.
2. If evidence is contradictory, it needs full discussion.
3. If the model is inappropriate, then this cannot be ignored.

Each of these possible issues and their recommended approach has been considered, and based on the qualitative and quantitative results, the models specified are accurate and appropriate. For a more detailed discussion of these results, we refer the reader to the **Conclusions** chapter below.

Chapter 5

Conclusions

This chapter finalises the research by summarising and addressing the results. We also propose areas for further research which are beyond the scope of this research.

5.1 Concluding the Results

The purpose of this thesis was to model and determine the mechanics of close-out netting. The research question is “What is the nature of close-out netting on bank portfolios?” From a high-level overview perspective, close-out netting is just a simple process of taking the difference between two counterparty exposure amounts. However, it is the stochastic volatility of daily FX rates and the multivariate nature of inter-bank netting that makes the purpose of this thesis much more difficult to research.

To address these issues, a modelling environment was established to measure the impact of close-out netting by incorporating the FX rates of five main currencies; USD/AUD, USD/CAD, USD/EUR, USD/GBP and USD/JPY. A banking context was chosen to have a more practical framework under which close-out netting is typically applied.

To realistically reproduce this environment, five GARCH(1,1) models were derived using actual FX rate data for one year and simulated out of sample, also for one year. Tests were then carried out on the resulting distributions to determine convergence, frequency, normality, closeness-of-fit to the distributions and other statistical metrics. Close-out netting was applied using a carefully specified netting framework or methodology. To ensure that the resulting netted time-series were not anomalies, the same process was repeated 250 times via MC simulation. A number of further statistical tests were applied to ensure the significance of the resulting im-

pact of close-out netting. These analyses showed that the results were not simulation anomalies but were consistent and robust.

Close-out netting was found to have a statistically significant impact on a portfolio of bank FX derivatives for all chosen FX rates except USD/JPY. This was found using a Two Sample t-Test. This initial result may be of no surprise, since taking the difference of two numbers is a significant process or result in itself. However, it is the value of this thesis to demonstrate that netting is far from a simple process by examining the nature of this exposure reduction. The fact that some FX currencies or banks may not necessarily be impacted in significant terms by netting has certainly not been widely documented in the literature.

Even this result is lined with certain caveats. Firstly, the techniques used to compare the distributions before and after netting assume one is comparing two Gaussian distributions, more particularly if the data sets are small. However, the netted distributions are not Gaussian; ranging from bimodal, truncated and extremely low variance distributions. To overcome the problems with the t-Test, the Signed Rank Two-Sample Wilcoxon Test was adopted, as such a test makes no assumptions about the nature of the two distributions. The results for this test essentially show that netting still has a significant impact, except for USD/JPY which is counter-intuitive. However, these Wilcoxon results were found to have less certainty than the t-Test results. We are compelled to conclude that there is no simple formula that can calculate how much exposure can be reduced by netting. The reason for this complexity lies not only in the stochastic volatility of daily FX rates, but also in the resulting multivariate nature of bilateral netting arrangements between multiple banks and the relative magnitude of these amounts. The netting formulas in **Chapter 3 - Modelling Methodology** and **Appendix A - Additional theoretical results** capture this complexity.

Emmons (1995) also pointed out that a system of banks would ultimately achieve zero credit risk if all deals are netted off due to a “sufficiently large number” of banks offsetting each other. Emmons pointed out that such a number may be as small as 100 banks. This thesis’ use of first 5 banks and then 4 banks suggests that the convergence observed may stabilise with far fewer banks than 100, say 20.

So far, these results have focussed on answering just the question of “how much” exposure is reduced by netting. We can now address more of the “how”. The modelling showed, through

rigorous simulation, that close-out netting was more impacted by some currencies than other currencies. This is a finding that is not so widely known and lead to the proposal of the MIN-NET Principal after MINimum NETting. Essentially this states that the amount of netting that occurs between any two counterparties is inversely proportional to the difference in pre-netted exposure amounts. To further explain this phenomenon, consider the schematic example in Figure 5.1 where two bank counterparties ¹ exposures are close-out netted in the two most common scenarios.

¹One of these counterparties is referred to as “A Bank” to highlight that the calculations have been measured with reference to that bank counterparty.

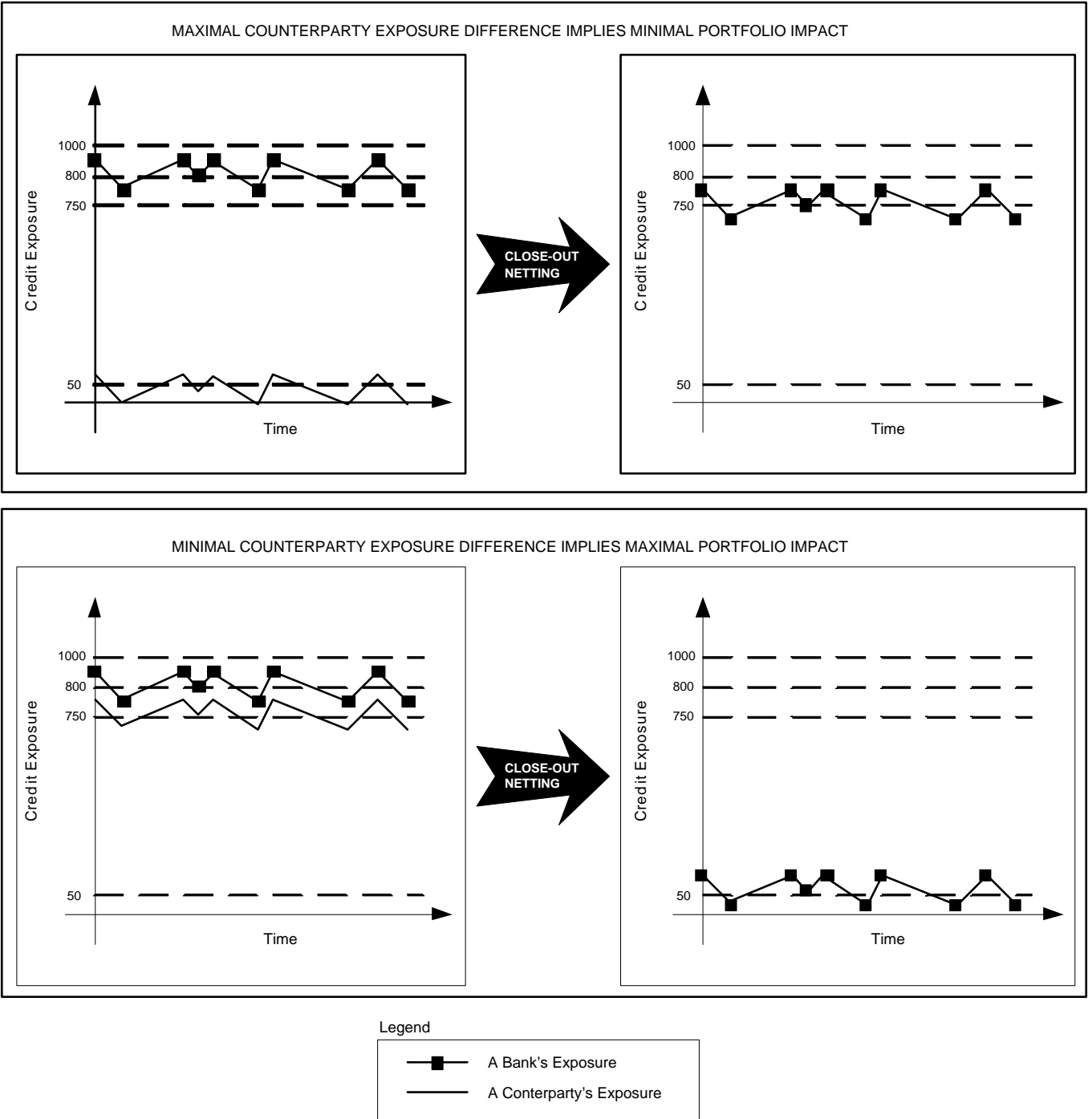


Figure 5.1: Schematic Example of Close-out Netting When the Exposures are Far Apart, and When they are Close Together

In Figure 5.1, a maximum exposure is set at \$1,000 and the minimum at \$0 with a mid-point of \$500. This is just to define simple reference points. Notice that in the first diagram, $\$800 - \$50 = \$750$ which is not too far from the mid-point, however in the second diagram, $\$800 - \$750 = \$50$ which is far from the mid-point. This implies that depending on the magnitudes of the credit exposures of the counterparties that are close-out netting, then the maximum impact to the Bank due to close-out netting is observed at the minimum counterparty exposures.

The MINNET Principal

$$\text{Maximum Portfolio Impact} \propto \frac{1}{\text{Minimum Counterparty Exposure}}. \quad (5.1)$$

It was demonstrated from the simulated distributions that close-out netting significantly reduced the mean -which is obvious. Although not present in these distributions, for certain currencies, close-out netting can significantly increase the variance -which is not so obvious (Bech, Madsen and Natrop, 2002). This implies that under certain conditions such as portfolio re-investment, close-out netting can increase the volatility of the bank's own portfolio. This occasional increase in variance means that banking and finance professionals that have implemented netting should now monitor their portfolio exposures and volatility more closely to be able to determine whether they are being exposed to greater levels of liquidity risk than expected. Such a peculiar disadvantage of netting is best expressed as "risk transfer". By this we mean that banks that are not heavily netted can have a significant disadvantage over other banks that are heavily netted, in that such banks have effectively absorbed or shouldered much of their counterparty's credit risk. Such an event would also be determined by the combined overall simultaneous effect of the stochastic volatility of daily FX rates at some point(s) in time.

Another effect of netting is that a certain amount of information about the portfolio exposure is lost due to its subtractive characteristics. This has a similar effect as when accountants aggregate low-level figures into high-level subtotals and totals. If the low-level figures are lost, then it is either very difficult or even impossible to drill down from these high-level figures. This is because as a bank nets off its portfolio against its counterparties, it introduces multivariate complexities into the portfolio that make it difficult for one to forecast future trends. Although it wasn't present in these results, netting can cause sudden reduction spikes or drops in the exposures down to zero. These structural breaks make it much more complex to model using

conventional techniques. Despite these netting difficulties, it is a small disadvantage compared to the overall reduction in exposure to credit default risk.

5.2 Addressing Possible Accounting Issues

The following issues have been identified as being concerns that either accountants may have or that the general public may have over the introduction of close-out netting and its impact.

- **How Does Close-out Netting Relate to Contra and Offset Entries in Accounting?**

To answer this question, a definition of a contra account is provided.

Definition 7.1 (Contra-Account)

A account that accumulates data and that is deducted/offset/netted against some other account. Furthermore, contra accounts can be in separate firms which can offset each other, such as when two entities are both debtors and creditors to each other (CCH Macquarie, 1990).

In light of this, close-out netting can be implemented in accounting systems by using contra accounts and offsetting or netting them as required.

- **Could Portfolio Reinvestment due to Close-out Netting be Considered as a Form of Market Manipulation?**

Portfolio reinvestment due to close-out netting could be negatively seen by certain investors as market manipulation by a bank to make its share price seem higher than its true value, as its credit default risk is minimised. Although such portfolio reinvestment is actually beneficial to the corresponding counter-party, its shareholders and its customers, unfortunately people's perceptions usually become their reality. This possible negative perception may lead some investors to contribute to a drop in share price. So long as the benefits of close-out netting are explained to these parties, then they should dispel such concerns.

- **Would Close-out Netting be Unfairly Applicable to Only Certain Banks?**

Firstly, close-out netting is available to any financial institution that lends and borrows money. It is not cost prohibitive or exclusive. Although the larger banks are better

equipped to implement close-out netting due to having larger legal and computing departments, the smaller banks can still enjoy the benefits of close-out netting. As mentioned above, it is in the best interests of all bank counterparties to close-out net. The fact that the larger banks will have more money that can be close-out netted is just a natural consequence of larger companies having greater economies of scale.

- **What, if any, are the Tax Implications of Close-out Netting?**

Due to close-out netting's relatively recent introduction, the Australian Taxation Office (ATO) has not yet released any new policies for the taxation of money freed up by close-out netting. The ATO is unlikely to tax close-out netted amounts because these are not genuine forms of income. However, governments can change their stances and the ATO could introduce a new tax on either the amount of money in capital reserves freed up by close-out netting or on portfolio reinvestment profits enabled by close-out netting. Furthermore, such a tax would effectively be taxing corporations at least three times; the first time when the asset is generated, the second when it is close-out netted, and the third when it is realised or liquidated. Nevertheless, due to the credit risk reduction and leveraging or gearing benefits of close-out netted amounts, even such a tax could still be worthwhile paying.

- **Does Close-out Netting Reduce Investor Confidence in Banks?**

As close-out netting frees up amounts of money in capital adequacy accounts and is reinvested, this can potentially increase the credit exposure risk within bank portfolios, and in particular market risk. If anything, there may be a risk of bad publicity over close-out netting if it can ever be given a negative bias. This risk though, is negligible given that banks that adopt close-out netting receive better reviews for the components that make up their overall credit risk rating. Although close-out alone is not usually enough to raise a bank's credit risk rating, it certainly is heavily encouraged by the Bank for International Settlements (BIS) in its Basel Pillar I and II reforms (Basel Committee, 2001). This in turn can only increase investor confidence.

5.3 Further Research

Close-out netting is an interesting and rich source of new research that can be studied further and enhanced in many directions. The following selected enhancements should uncover further truths about close-out netting that cannot be adequately addressed in this thesis.

5.3.1 Determining the Relationship Between Close-out Netting and Capital Adequacy

The Bank for International Settlements (BIS) allows a bank's capital adequacy reserves to be reduced so long as it reduces its credit risk exposure, such as by using close-out netting. Just to what extent a bank should lower these reserves depends on the amount close-out netted. To determine the exact relationship between levels of close-out netting and levels of capital reserves is a complex task because one would have to further analyse what was happening to the portfolio at least at the asset group level during close-out netting. A portfolio VaR with and without netting would need to be formulated and estimated before a relationship could be found with capital adequacy. The stochastic volatility of FX rates would force the calculations to be processed over time series data to determine convergence, drift and other stochastic metrics.

5.3.2 Collateralisation Under Uncertainty

Relatively recently in mid 2003, Australian banks began taking the next step forward after the full implementation of close-out netting, i.e. collateralisation ². It is one thing to be able to close-out net contracts and record them as if the counterparty was bankrupt. To go the next step further though, and collateralise one's assets against one's counterparty's assets makes close-out netting all the more worthwhile. How this begins to become complex is that a bank is willing to only collateralise some of its portfolio contracts with only some of their counter-parties portfolio contracts. This is not to mention that at a higher level, certain asset groups are cross-collateralised ³ with other asset groups. The more cross-collateralisation the better, as there are more assets to seize in the case of default. Care must be taken though not to count these assets more than once, otherwise the benefits of netting may be overstated. These refinements would significantly expand the required simulation space and the scope of this research.

5.3.3 Incrementally Increasing the Number of Counterparties

One could begin by further analysing the effects of close-out netting on not just five counterparties but incrementally up to say the world's top one hundred banks. This would enable one to see more clearly the underlying mechanics of close-out netting in relation to the increasing number of counterparties, especially when various convergence patterns emerge. This form of

²Collateralisation can be defined as "Assets pledged by a borrower to secure a loan or other credit, and subject to seizure in the event of default, and is also known as security" (www.InvestorWords.com).

³Cross-collateralisation can be defined as "When collateral for one loan is also serving as collateral for other loans" (www.InvestorWords.com).

further research would begin by utilising six banks, then seven and so on. This thesis' use of five counterparties though is more than adequate to assess the nature of close-out netting.

5.3.4 Varying Portfolio Asset Group Concentrations

Yet another possible enhancement could be to determine whether close-out netting impacts are sensitive to the weighting factors in a portfolio's asset group concentration. This thesis has assumed for simplicity and generality that the banks' portfolios were composed of one asset group that depends on FX rates (just the raw FX rate itself). If other asset groups were introduced, each with varying reliance on FX rates, then a less theoretical approach could be adopted which may lead to the isolation of various "netting thresholds". Such thresholds would be upper bounds for the benefits of close-out netting. A related area is that of asset group correlations, measured by covariance matrices. Such an enhancement would appear to be easily incorporated into the existing matrix netting notation.

5.3.5 Analysing Stages of Economic Cycles

Depending on what stage a bank may be throughout an economic cycle, banks would structure their portfolios differently during a boom period than during a recession. The 2004 and 2005 FX rate data input into the GARCH(1,1) models are taken from a period of relative international growth. The difficulty in simulating over say, a 20 year period to ensure all stages of a cycle are explored is that one would increasingly lose forecast accuracy over time. One approach for further research could be to reproduce the results of this thesis for each of the four stages of an economic cycle.

5.3.6 Enhancing Close-out Netting Research via Copulas

Whilst VaR is a reasonable measure of a portfolio's exposure to credit default risk, it relies heavily on correlations. Although most of the close-out netting data is distributed about a relatively low risk mean, it is in the tail -which is skewed to the right -where most of the high risk contracts lie. These types of datasets prevent conventional correlation methods from being applied due to the uniqueness of these types of distributions and that most methods focus on the bulk of the data and not on the few exceptions.

Embrechts, Lindskog *et. al.* (2001) have used copulas to model these correlations. The theory of copulas is an abstract and powerful theory that looks at the similarities between different

probability distributions by looking at their underlying components in an n -dimensional space. Copulas are thus able to answer questions that are either very difficult or impossible to solve using conventional techniques. The use of copulas would enable close-out netting research to better compare the portfolio model distributions with and without close-out netting, in cases where the distributions being compared are highly skewed. The resulting netted distributions certainly are not standard. Such use of skewed distributions, in particular log-normal distributions in the context of Figure B.1, would extend and refine the existing research on close-out netting.

5.3.7 Generating Quasi-Random Numbers for Quasi-Monte Carlo Simulation

We plot 500 pseudo-random numbers in Figure 5.2 as part of a Monte Carlo simulation.

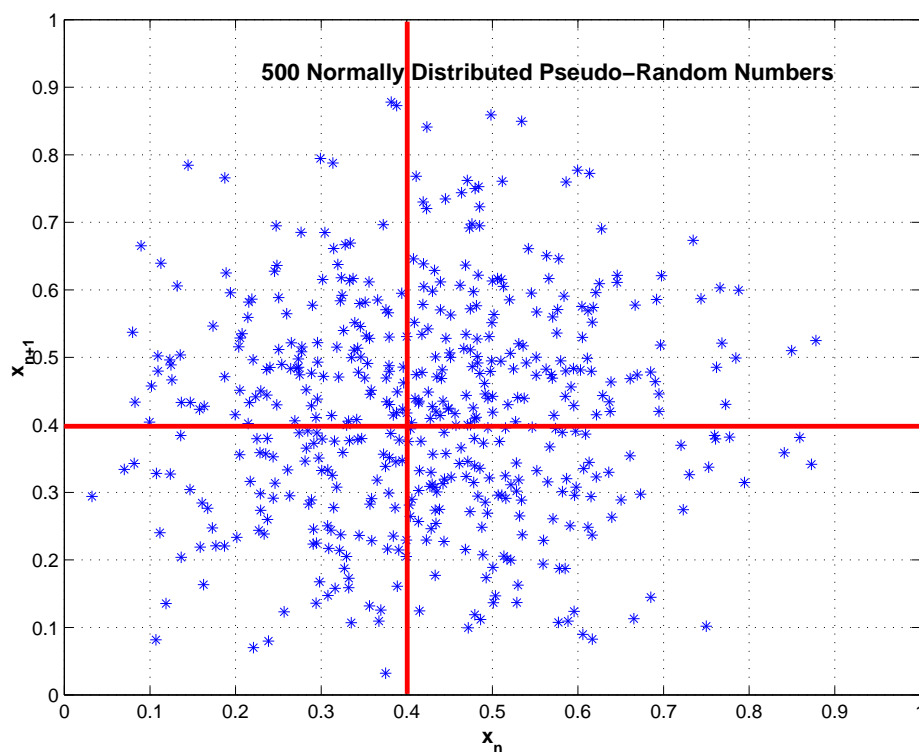


Figure 5.2: 500 R250 Normalised 2D-Plot Pairs (x_n, x_{n+1}) Plot

Taking a closer look at Figure 5.2, one can see that there are certain regions that have a much greater concentration of points than in other regions. These concentrations are a property of pseudo-random numbers that is not desirable for certain model simulations due to these numbers not covering as much of the variation space. However, real-life distributions have these concentrations and so they shouldn't automatically be eliminated from the modelling. One possible

hypothesis may be that the minimisation of these concentrations may give rise to more robust results.

If these concentrations need to be reduced for one's simulations, then one available technique is the use of quasi pseudo-random numbers, or just quasi-random numbers. Quasi-random numbers can extend this research on close-out netting by facilitating the creation of alternate models that are not based on standard pseudo-random numbers.

For further details on quasi-random numbers, please refer to **Appendix C - Further Quasi-Random Number Details**.

Bibliography

- [1] Abad J., and Hurd T. (2003) *An Error Formula for Monte Carlo Based Portfolio Optimization*. McMaster University Press, Canada.
- [2] Al-Mharmah H. (1998) *Average Performance Of Quasi Monte Carlo Methods For Global Optimization*. University of Jordan.
- [3] Algorithmics (2000) *Risk Analytics in Risk Watch; Version 3.2*. Ontario, Algorithmics Inc.
- [4] Algorithmics (2001) *The Regulatory Capital Treatment of Credit Exposures Arising from Derivative Transactions*. Ontario, Algorithmics Inc.
- [5] Allsopp P. (1996) *Settlement Risk In Foreign Exchange Transactions*. Basel, Committee on Payment and Settlement Systems.
- [6] Allsopp P. (1989) *Report on Netting Schemes - (the Angell Report)*. Basel, Committee on Payment and Settlement Systems.
- [7] Andersen T. and Bollerslev T. (1997) *Intraday Periodicity and Volatility Persistence in Financial Markets*. Journal of Empirical Finance 4, 115-158.
- [8] Andersen T., Bollerslev T., Diebold F. and Labys P. (1999) *Exchange Rate Returns Standardized by Realized Volatility are (Nearly) Gaussian*. National Science Foundation.
- [9] APRA (2000) *APRA Guidance Note, AGN 112.3 ("Netting Guidance Note")*. Sydney, Australian Prudential Regulation Authority (APRA).
- [10] APRA (1999) *Prudential Statement C1; Capital Adequacy of Banks*. Sydney, Australian Prudential Regulation Authority (APRA).
- [11] APRA (1998) *Prudential Statement C3; Market Risk*. Sydney, Australian Prudential Regulation Authority (APRA).
- [12] APRA (1989) *Prudential Statement E1; Supervision of Bank's Large Credit Exposure*. Sydney, Australian Prudential Regulation Authority (APRA).

- [13] APRA (2000) *APRA Submission to the Basel Committee on Banking Supervision -A New Capital Adequacy Framework*. Australian Prudential Regulation Authority (APRA).
- [14] Australian Parliament (1998) *Payment Systems and Netting Act 1998* (“Netting Act”). Australian Parliament.
- [15] Bailey R. (2001) *Economics of Financial Markets*. Department of Economics, University of Essex.
- [16] Basel Committee (1991) *Credit Risk Modelling: Current Practices and Applications*. Basel, Bank for International Settlements.
- [17] Basel Committee (2001) *The Internal Ratings-Based Approach*. Basel, Bank for International Settlements.
- [18] Basel Committee (2001) *The New Basel Capital Accord*. Basel, Bank for International Settlements.
- [19] Basel Committee (2001) *The Standardised Approach to Credit Risk*. Basel, Bank for International Settlements.
- [20] Basel Committee (2000) *Proposals for the management of Credit Risk*. Basel, Bank for International Settlements.
- [21] Bech M., Madsen B. and Natrop L. (2002) *Systemic Risk in the Danish Interbank System*. Danmarks Nationalbank Working Papers.
- [22] Brent R. and Zimmermann P. (2003) *Random Number Generators With Period Divisible by a Mersenne Prime*. Oxford University Computing Laboratory.
- [23] Berkowitz J. and O’Brien J. (2002) *How Accurate Are Value-at-Risk Models at Commercial Banks?* The Journal of Finance, Volume 57pp. 1093.
- [24] Berndt E., Hall B., Hall R., and Hausman J. (1974) *Estimation and Inference in Nonlinear Structural Models*. Annals of Economic and Social Measurement.
- [25] Black F. and Scholes M. (1973) *The Pricing of Options and Corporate Liabilities*. The Journal of Political Economy, May-June.
- [26] Bollerslev T. (1986) *Generalized Autoregressive Conditional Heteroskedasticity*. Journal of Econometrics 31: 307-327.
- [27] Baillie R. and Bollerslev T. (1991) *Intra-Day and Inter-Market Volatility in Foreign Exchange Rates*. Review of Economic Studies 58: 565-585.

- [28] Bollerslev T., Chou R. and Kroner K. (1992) *ARCH Modeling in Finance: A Review of the Theory and Empirical Evidence*. Journal of Econometrics, 52, 5-59.
- [29] Bollerslev T., Baillie R. and Mikkelsen H. (1996) *Fractionally Integrated Generalized Autoregressive Conditional Heteroskedasticity*. Journal of Econometrics 74, 3-30.
- [30] Brown C. (1995) *Accounting Expert Systems Applications*. Oregon State University.
- [31] Cantor R. and Falkenstein E. (2001) *Testing for Rating Consistency in Annual Default Rates*. Moody's Investors Service.
- [32] Chakravorti S. (2000) *Analysis of Systemic Risk in Multilateral Net Settlement Systems*. Journal of International Financial Markets, Institutions and Money, Elsevier, vol. 10(1), pp. 9-30.
- [33] Corrigan D., et. al. (1999) *REPO -The Ultimate Guide*. Pearls of Wisdom Publishing Ltd.
- [34] Credit Suisse First Boston (1997) *CreditRisk+ -A Credit Risk Management Framework*. Credit Suisse First Boston International.
- [35] Credit Suisse First Boston (CSFB) (1997) *CreditRisk+: A Credit Risk Management Framework*. Technical Document, Credit Suisse First Boston International.
<http://www.csfb.com/creditrisk>
- [36] Crouchy M., Galai D. and Mark R. (2001) *Risk Management*. McGraw Hill, New York.
- [37] D'Agostino R. (1970) *Transformation to Normality of the Null Hypothesis of g_1* . Biometrika, **57**, 679-681.
- [38] De Brandt O. and Hartmann P. (2000) *Systemic Risk: A Survey*. Working Paper No. 35, European Central Bank, Frankfurt.
- [39] Delisio J., McGowan M. and Hamscher W. (1993) *PLANET: An Expert System for Audit Risk Assessment and Planning*. Fifth International Conference on Intelligent Systems in Accounting, Finance and Management.
- [40] Dembo R. et. al. (2000) *Mark-to-Future: A Framework for Measuring Risk and Reward*. Ontario, Algorithmics Inc.
- [41] Duffie D. and Singleton K. (1999) *Modeling the Term Structures of Defaultable Bonds*. Review of Financial Studies, 12, 687-720.
- [42] Ellis L. (1999) *Business Finance*. Melbourne, Monash University Press.
- [43] Embrechts P., Lindskog F. et. al. (2001) *Modelling Independance with Copulas and Applications to Risk Management*. Zurich, Eidgenossische Technische Hochschule Zentrum.

- [44] Emmons W. (1995) *Interbank Netting Agreements and the Distribution of Bank Default Risk*. Working Paper, Federal Reserve Bank of St. Louis.
- [45] Embrechts P., Resnick S. *et. al.* (1997) *Extreme Value Theory as a Risk Management Tool*. First presented by the first author as “Extremes and Insurance” at the XXVIII International ASTIN Colloquium in Cairns, Australia.
- [46] Engle R. (1982) *Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of U.K. Inflation*. *Econometrica* 50: 987-1008.
- [47] Estrella A. and Hendricks D. (1993) *Netting and Potential Exposure: A Simple Statistical Illustration*. Federal Reserve Bank of New York, unpublished technical working paper.
- [48] ERisk (1994) *Limit Utilization: Timing and Netting*. Published in ERisk Magazine by OWC Credit Inc.
- [49] Faure H. (1982) *Discrepance de Suites Associees a un Systeme de Numeration (en Dimensions)*. *Acta Arithmetica*. 41:337-351.
- [50] Fermanian D. and Scaillet O. (2003) *Sensitivity Analysis of VaR and Expected Shortfall*. International Centre for Asset Management and Engineering.
- [51] Fernandes M. (1998) *Forecasting Financial Crashes: A Catastrophe Theory Approach*. Université Libre de Bruxelles.
- [52] Figlewski S. (1997) *Forecasting Volatility*. Financial Markets, Institutions & Instruments.
- [53] Finger C. (2000) *A Comparison of Stochastic Default Risk Models*. New York, The Risk-Metrics Group.
- [54] Fishman G. (1996) *Monte Carlo: Concepts, Algorithms, and Applications -2nd Edition*. Springer Verlag.
- [55] FRBNY (2000) *Calculating Yields on Treasury Bills for Comparison Purposes*. Federal Bank of New York Publications.
- [56] Gelfand A. and Smith A. (1990) *Sampling-Based Approaches to Calculating Marginal Densities*. *Journal of American Statistical Association*, 85 pp. 398409.
- [57] Gizycki M. and Gray B. (1994) *Default Risk and Derivatives: An Emperical Analysis of Bilateral Netting*. Working Paper, Reserve Bank of Australia.
- [58] Gopikrishnan P., Plerou V. *et. al.* (1999) *Scaling of the Distribution of Fluctuations of Financial Market Indices*. www.lanl.gov/cond-mat/9905305.

- [59] Gray B. (1999) *Ammendments to Capital Adequacy Guidelines -Netting*. APRA Media Release, September.
- [60] Gupton G. (1997) *The New Talk of the Town -CreditMetrics, A Credit Value-at-Risk Approach*. The Journal of Lending and Credit Risk Management, Vol. 79, No. 12, J.P. Morgan, pp. 271-299.
- [61] Halton J. (1960) *On the Efficiency of Certain Quasirandom Sequences of Points in Evaluating Multidimensional Integrals*. Numerische Mathematik, 2:84-90.
- [62] Hammersley J. and Handscomb D. (1961) *Monte Carlo Methods*. Methuen, London.
- [63] Hansen P. (2001) *An Unbiased and Powerful Test for Superior Predictive Ability*. <http://chico.pstc.brown.edu/phansen>.
- [64] Hansen P. and Lunde A. (2001) *A Forecast Comparison of Volatility Models: Does Anything Beat a GARCH (1,1)?*. Brown University.
- [65] Haselgrove C. (1961) *A Method for Numerical Integration*. Mathematics of Computation, **15** 323-337.
- [66] Holton G. (1995) *Closed Form Value at Risk*. Massachusetts, Contingency Analysis Co.
- [67] Holton G. (2001) *Risk Measures*. Boston, Contingency Analysis Inc.
- [68] Huisman R. *et. al.* (1999) *Asset Allocation in a Value-at-Risk Framework*. Erasmus University Rotterdam, The Netherlands.
- [69] Hull J. (2000) *Options, Futures & Other Derivatives; Fourth Edition*. Prentice-Hall, Inc.
- [70] Ieda A. *et. al.* (2000) *A Simplified Method for Calculating the Credit Risk of Lending Portfolios*. Institute of Monetary and Economic Studies, Bank of Japan.
- [71] International Financial Risk Institute (IFRI) (2000) *Overview: Netting and Insolvency*.
- [72] ISDA (1999) *A New Capital Adequacy Framework: Comments on the Consultative Paper Issued by the Basel Committee on Banking Supervision*. International Swaps and Derivatives Association (ISDA).
- [73] J.P. Morgan (1997) *CreditMetrics: The Benchmark for Understanding Credit Risk - Technical Document*. J.P. Morgan Pty. Ltd.
- [74] Joe H. (1997) *Multivariate Models and Dependence Concepts*. London, Chapman and Hall.
- [75] Kloeden P. and Platen E. (1992) *Numerical Solution of Stochastic Differential Equations*. Springer Verlag.

- [76] Kealhofer S. (1998) *Portfolio Management of Default Risk (Portfolio Manager)*. KMV Corporation.
- [77] Lamfalussy (1990) *Report of the Committee on Interbank Netting Schemes - (the Lamfalussy Report)*. Basel, Committee on Interbank Netting Schemes.
- [78] Lemieux C. and L'Ecuyer P. (2001) *On the Use of Quasi-Monte Carlo Methods in Computational Finance*. Lecture Notes in Computer Science, Springer Verlag.
- [79] Levy A. and Clarke M. (2000) *Modelling Derivative Credit Risk in The Prescence of Collateral and Netting*. J. P. Morgan Securities Inc.
- [80] Lyuu Y. (2002) *Financial Engineering and Computation; Principals, Mathematics, Algorithms*. Cambridge University Press.
- [81] Mandelbrot B. (1963) *The Variation of Certain Speculative Prices*. Journal of Business.
- [82] Marphatia A. and Tiwari N. (2000) *Risk Management in the Financial Services Industry -An Overview*. Institute of Electrical and Electronics Engineers (IEEE) Bombay.
- [83] Marsaglia G. (1968) *Random Numbers Fall Mainly in the Planes*. Proceedings from the National Academy of Science, Volume 61.
- [84] Marsaglia G. and Tsang W. (1984) *The Ziggurat Method for Generating Random Numbers*. SIAM Journal of Scientific and Statistical Programming, Volume 5.
- [85] Matsumoto M. and Nishimura T. (1998) *Mersenne Twister: A 623-Dimensionally Equidistributed Uniform Pseudo-Random Number Generator*. ACM Press.
- [86] Medova E. (2000) *Operational Risk Measures and Bayesian Simulation for Capital Allocation*. Centre for Financial Research, University of Cambridge.
- [87] Macquarie (1991) *The CCH Macquarie Dictionary of Accounting*. CCH Press.
- [88] Merton R. (1974) *Theory of Rational Option Pricing*. Bell Journal of Economics and Management Science
- [89] Michaux F. (2001) *Australian Interest Rate Criteria for Structured Finance Ratings*. Structured Finance: Standard & Poors.
- [90] Morris S. (2000) *Market Risk with Interdependent Choice*. Yale University Press.
- [91] NICHSR (2005 United States National Information Center on Health Services Research and Health Care Technology.
- [92] Neely C. and Weller P. (2001) *Predicting Exchange Rate Volatility: Genetic Programming vs. GARCH and RiskMetrics*. FEDERAL RESERVE BANK OF ST. LOUIS.

- [93] Nelsen R. (1999) *An Introduction to Copulas (Lecture Notes in Statistics, Volume 139)*. Springer Verlag.
- [94] Nelson D. (1991) *Conditional Heteroscedasticity in Asset Returns: A New Approach*. *Econometrica*, 59, 347-370.
- [95] Niederreiter H. (1992) *Random Number Generation and Quasi-Monte Carlo Methods*. SIAM, CBMS-NSF Regional Conference Series in Applied Mathematics, 63.
- [96] Nishiguchi K., *et. al.* (1998) *Capital Allocation and Bank Management Based on the Quantification of Credit Risk*. Federal Reserve Bank of New York (FRBNY), October.
- [97] Ong M. (1999) *Modelling Credit Risk Internally*. London, Risk Publications.
- [98] Ostaszewski K. and Rempala G. (2000) *Parametric and Nonparametric Bootstrap in Actuarial Practice*. University of Louisville.
- [99] Padoa-Schioppa T. (1999) *Licensing Banks: Still Necessary?* European Central Bank.
- [100] Phelan K. and Alexander C. (1999) *Credit Models: Different Strokes*. *Credit Risk Magazine*, October.
- [101] Premaratne G. and Bera A. (2005) *A Test for Symmetry with Leptokurtic Financial Data*. *Journal of Financial Econometrics*, p169-187.
- [102] Ramamoorti S., Bailey A. Jr. and Traver R. (1998) *Neural Network Modelling of Risk Assessment in Internal Auditing*. University of Illinois.
- [103] Ramaswamy S. (1998) *Portfolio Selection Using Fuzzy Decision Theory*. Basel, Bank for International Settlements.
- [104] Rastas K., Karinen J., *et. al.* (1998) *Development of an Analysis System for Company's Financial Statement Analysis*. Balance Consulting Inc.
- [105] Rosen D. (2000) *An Integrated Market and Credit Risk Framework*. Ontario, Algorithmics Inc.
- [106] Rubinstein R. (1981) *Simulation and the Monte Carlo Method*. John Wiley & Sons.
- [107] Sincich T. (1990) *Business Statistics by Example*. Macmillan Publishing Co.
- [108] Shapiro A. and Homem-de-Mello T. (2000) *On the Rate of Convergence of Optimal Solutions of Monte Carlo Approximation of Stochastic Programs*. *Society for Industrial and Applied Mathematics (SIAM) Journal on Optimization*, Volume 11, [1].
- [109] Sobol I. (1967) *On the Distribution of Points in a Cube and the Approximate Evaluation of Integrals*. *U.S.S.R. Computational Mathematics and Mathematical Physics*. 7(4):86-112.

- [110] Steiner R. (1998) *Mastering Financial Calculations: A step-by-step guide to the mathematics of financial market instruments*. Great Britan, Pitman Publishing.
- [111] Stoll H. and Whaley R. (1993) *Futures and Options: Theory and Applications*. Ohio, South Western Publishing Company.
- [112] Study Group on OTC Derivatives (1998) *OTC Derivatives: Settlement Procedures and Counterparty Risk Management*. Basel, Bank for International Settlements.
- [113] Stumbles R. and Kerr E. (1998) *Banking Law - Issue 107, April*. Mallesons Stephen Jaques Pty. Ltd.
- [114] Ullrich C. (2005) *Estimation of Continuous-Time Models of Foreign Exchange Volatility*. University of Maryland press.
- [115] Vasicek O. (1987) *KMV Portfolio Manager Model*. KMV Corporation.
- [116] Vlaar P. (2000) *Value at Risk Models for Dutch Bond Portfolios*. Journal of Banking & Finance.
- [117] Wahrenburg M. (1997) *Measuring and Managing the Credit Exposure of Derivatives Portfolios*. University of Köln.
- [118] White H. (2000) *A Reality Check for Data Snooping*. Econometrica, 68, 10971126.
- [119] Wilmott P. (2000) *Paul Wilmott on Quantitative Finance, Volume 1 & 2 (of 2)*. England, John Wiley & Sons.
- [120] Wilson T. (1997) *CreditPortfolioView*. McKinsey and Company.
- [121] Yang L. (2003) *A Semiparametric GARCH Model for Foreign Exchange Volatility*. Michigan State University.

Appendix A

Additional Theoretical Results

A.1 Example of Three Banks

Take three Banks x_1 , x_2 and x_3 . Their exposures through bilateral netting agreements can be shown schematically in Figure A.1.

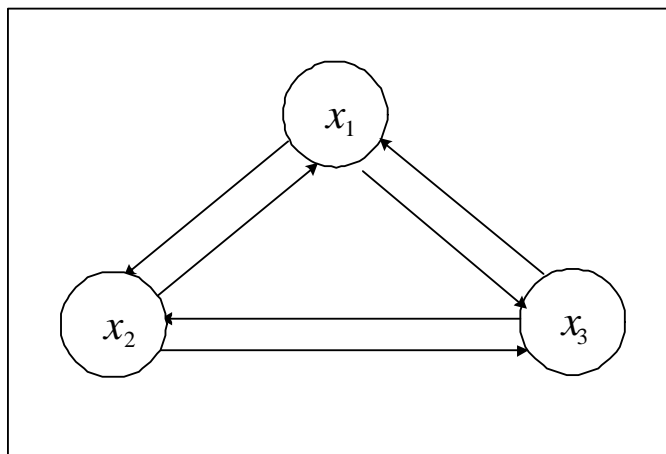


Figure A.1: Exposures Between Three Banks

Suppose that in our three Bank example of Bank x_1 , Bank x_2 and Bank x_3 , that at some point in time t ,

$$e_{1,2}(t) = \$50,000,000$$

$$e_{1,3}(t) = \$30,000,000$$

$$e_{2,1}(t) = \$20,000,000$$

$$e_{2,3}(t) = \$40,000,000$$

$$e_{3,1}(t) = \$10,000,000$$

$$e_{3,2}(t) = \$60,000,000.$$

Following on from our netting formulation of Equation 3.7, the number of Banks n is $n = 3$, giving,

Without Netting

$$\begin{aligned} E &= \left(\begin{bmatrix} 0 & e_{1,2}(t) & e_{1,3}(t) \\ e_{2,1}(t) & 0 & e_{2,3}(t) \\ e_{3,1}(t) & e_{3,2}(t) & 0 \end{bmatrix} \right) \\ &= \begin{bmatrix} 0 & \$50,000,000 & \$30,000,000 \\ \$20,000,000 & 0 & \$40,000,000 \\ \$10,000,000 & \$60,000,000 & 0 \end{bmatrix}. \end{aligned}$$

With Netting

$$\begin{aligned}
N(E) &= N \left(\begin{bmatrix} 0 & e_{1,2}(t) & e_{1,3}(t) \\ e_{2,1}(t) & 0 & e_{2,3}(t) \\ e_{3,1}(t) & e_{3,2}(t) & 0 \end{bmatrix} \right) \\
&= \begin{bmatrix} 0 & \max\{e_{1,2}(t) - e_{2,1}(t), 0\} & \max\{e_{1,3}(t) - e_{3,1}(t), 0\} \\ \max\{e_{2,1}(t) - e_{1,2}(t), 0\} & 0 & \max\{e_{2,3}(t) - e_{3,2}(t), 0\} \\ \max\{e_{3,1}(t) - e_{1,3}(t), 0\} & \max\{e_{3,2}(t) - e_{2,3}(t), 0\} & 0 \end{bmatrix} \\
&= \begin{bmatrix} 0 & \max\{\$50,000,000 - \$20,000,000, 0\} & \max\{\$30,000,000 - \$10,000,000, 0\} \\ \max\{\$20,000,000 - \$50,000,000, 0\} & 0 & \max\{\$40,000,000 - \$60,000,000, 0\} \\ \max\{\$10,000,000 - \$30,000,000, 0\} & \max\{\$60,000,000 - \$40,000,000, 0\} & 0 \end{bmatrix} \\
&= \begin{bmatrix} 0 & \$30,000,000 & \$20,000,000 \\ 0 & 0 & 0 \\ 0 & \$20,000,000 & 0 \end{bmatrix}.
\end{aligned}$$

Hence, under netting, Bank x_1 has reduced its exposure to Bank x_2 down to \$30,000,000 and its exposure to Bank x_3 down to \$20,000,000. Bank x_2 no longer has an exposure to Bank x_1 nor Bank x_3 . Bank x_3 no longer has an exposure to Bank x_1 and has reduced its exposure to Bank x_2 down to \$20,000,000.

A.2 Illustration of Four and Five Banks

By looking at Equation 3.7, we see that it is already generalised for n Banks. Although there is not much point in repeating the example calculations for more than three Banks, it is at least worthwhile illustrating the multivariate complexity and number of bilateral netting agreements that exist between these number of banks. These have been illustrated in Figure A.2 and Figure A.3 below.

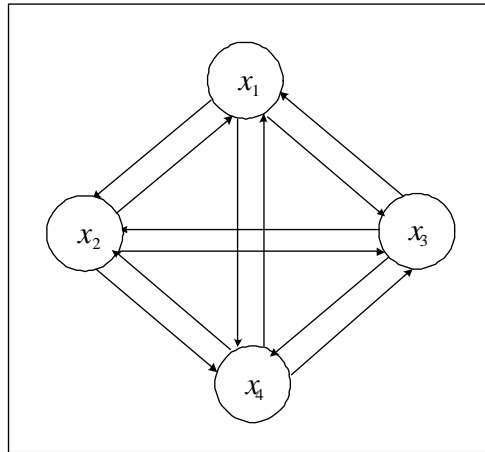


Figure A.2: Exposures Between Four Banks

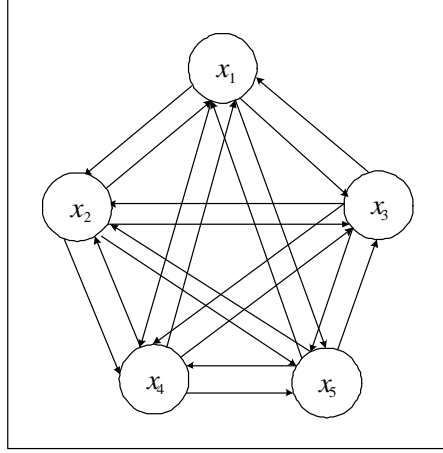


Figure A.3: Exposures Between Five Banks

It is also worthwhile noting that in general, the total number of bilateral netting contracts or agreements between n banks is $\frac{n(n-1)}{2}$.

A.3 Vector-Sequence Notation

Vector-sequence notation allows matrices to be more concisely expressed as a sequence of vectors. We can extend from the netting model by first mentioning that Equation 3.6 for the exposure without netting can be expressed as,

$$E = \begin{cases} \{e_{i,j}(t)\}_{i=1}^n, & \forall i \neq j, \quad i, j \in \{1, \dots, n\} \\ 0, & \forall i = j, \quad i, j \in \{1, \dots, n\} \end{cases}. \quad (\text{A.1})$$

Correspondingly, Equation 3.7 for the exposure with netting can be expressed as,

$$N(E) = \begin{cases} \{\max\{e_{i,j}(t) - e_{i,j}(t), 0\}\}_{i=1}^n, & \forall i \neq j, \quad i, j \in \{1, \dots, n\} \\ 0, & \forall i = j, \quad i, j \in \{1, \dots, n\} \end{cases}. \quad (\text{A.2})$$

A.4 Bank's Total Exposure

So far, the netted exposure matrix $N(E)$ has calculated the exposure each Bank i has to Bank j after netting. However, it is also useful to determine Bank i 's total exposure across all other banks, both with and without netting. Another way of expressing this is to determine Bank i 's multilateral exposure both with and without netting. We then define a Bank's Total exposure operator $T(*)$ for without netting in matrix notation as shown below,

Without Netting

$$\begin{aligned}
T(E) &= T \left(\begin{bmatrix} 0 & e_{1,2}(t) & \cdots & \cdots & e_{1,n}(t) \\ e_{2,1}(t) & 0 & \ddots & & \vdots \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & & \ddots & 0 & e_{n-1,n}(t) \\ e_{n,1}(t) & \cdots & \cdots & e_{n,n-1}(t) & 0 \end{bmatrix} \right) \\
&= \begin{bmatrix} 0 & e_{1,2}(t) & \cdots & \cdots & e_{1,n}(t) \\ e_{2,1}(t) & 0 & \ddots & & \vdots \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & & \ddots & 0 & e_{n-1,n}(t) \\ e_{n,1}(t) & \cdots & \cdots & e_{n,n-1}(t) & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \\ 1 \end{bmatrix} \\
&= \begin{bmatrix} e_{1,2}(t) + e_{1,3}(t) + \cdots + e_{1,n}(t) \\ e_{2,1}(t) + e_{2,3}(t) + \cdots + e_{2,n}(t) \\ \vdots \\ e_{n-1,1}(t) + e_{n-1,2}(t) + \cdots + e_{n-1,n-1}(t) \\ e_{n,1}(t) + e_{n,2}(t) + \cdots + e_{n,n-1}(t) \end{bmatrix}. \quad (A.3)
\end{aligned}$$

We define a Bank's Total Exposure with netting in matrix notation as shown below,

With Netting

$$\begin{aligned}
T(N(E)) &= T \left(\begin{bmatrix} 0 & \max\{e_{1,2}(t) - e_{2,1}(t), 0\} & \dots & \dots & \max\{e_{1,n}(t) - e_{n,1}(t), 0\} \\ \max\{e_{2,1}(t) - e_{1,2}(t), 0\} & 0 & \ddots & & \vdots \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & & \ddots & 0 & \max\{e_{n-1,n}(t) - e_{n,n-1}(t), 0\} \\ \max\{e_{n,1}(t) - e_{1,n}(t), 0\} & \dots & \dots & \max\{e_{n,n-1}(t) - e_{n-1,n}(t), 0\} & 0 \end{bmatrix} \right) \\
&= \begin{bmatrix} 0 & \max\{e_{1,2}(t) - e_{2,1}(t), 0\} & \dots & \dots & \max\{e_{1,n}(t) - e_{n,1}(t), 0\} \\ \max\{e_{2,1}(t) - e_{1,2}(t), 0\} & 0 & \ddots & & \vdots \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & & \ddots & 0 & \max\{e_{n-1,n}(t) - e_{n,n-1}(t), 0\} \\ \max\{e_{n,1}(t) - e_{1,n}(t), 0\} & \dots & \dots & \max\{e_{n,n-1}(t) - e_{n-1,n}(t), 0\} & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \\ 1 \end{bmatrix} \\
&= \begin{bmatrix} \max\{e_{1,2}(t) - e_{2,1}(t), 0\} + \max\{e_{1,3}(t) - e_{3,1}(t), 0\} + \dots + \max\{e_{1,n}(t) - e_{n,1}(t), 0\} \\ \max\{e_{2,1}(t) - e_{1,2}(t), 0\} + \max\{e_{2,3}(t) - e_{3,2}(t), 0\} + \dots + \max\{e_{2,n}(t) - e_{n,2}(t), 0\} \\ \vdots \\ \max\{e_{n-1,2}(t) - e_{2,n-1}(t), 0\} + \max\{e_{n-1,3}(t) - e_{3,n-1}(t), 0\} + \dots + \max\{e_{n-1,n}(t) - e_{n,n-1}(t), 0\} \\ \max\{e_{n,1}(t) - e_{1,n}(t), 0\} + \max\{e_{n,2}(t) - e_{2,n}(t), 0\} + \dots + \max\{e_{n,n-1}(t) - e_{n-1,n}(t), 0\} \end{bmatrix} .
\end{aligned} \tag{A.4}$$

Using the vector-sequence notation of Equation A.1 and Equation A.2, we can rewrite Equation A.3 and Equation A.4 as follows,

Bank's Total Exposure Without Netting in Vector-Sequence Notation

$$T(E) = \begin{cases} \left\{ \sum_{j=1}^n e_{i,j}(t) \right\}_{i=1}^n, & \forall i \neq j, \quad i, j \in \{1, \dots, n\} \\ 0, & \forall i = j, \quad i, j \in \{1, \dots, n\} \end{cases} \quad (A.5)$$

Bank's Total Exposure With Netting in Vector-Sequence Notation

$$T(N(E)) = \begin{cases} \left\{ \sum_{j=1}^n \max\{e_{i,j}(t) - e_{j,i}(t), 0\} \right\}_{i=1}^n, & \forall i \neq j, \quad i, j \in \{1, \dots, n\} \\ 0, & \forall i = j, \quad i, j \in \{1, \dots, n\} \end{cases} \quad (A.6)$$

A.5 Netting System's Total Exposure

Finally, it is worthwhile to define a Netting System's Total Exposure $S(*)$ as the sum of its multilateral exposures, giving,

Netting System's Total Exposure Without Netting in Vector-Sequence Notation

$$S(T(E)) = \begin{cases} \sum_{i=1}^n \left[\sum_{j=1}^n e_{i,j}(t) \right], & \forall i \neq j, \quad i, j \in \{1, \dots, n\} \\ 0, & \forall i = j, \quad i, j \in \{1, \dots, n\} \end{cases} \quad (A.7)$$

Netting System's Total Exposure With Netting in Vector-Sequence Notation

$$S(T(N(E))) = \begin{cases} \sum_{i=1}^n \left[\sum_{j=1}^n \max\{e_{i,j}(t) - e_{j,i}(t), 0\} \right], & \forall i \neq j, \quad i, j \in \{1, \dots, n\} \\ 0, & \forall i = j, \quad i, j \in \{1, \dots, n\} \end{cases} \quad (A.8)$$

This is useful because if one can measure a System's total exposure, both with and without netting, then one can determine whether or not netting has reduced that System's total exposure, and whether or not some Banks have had increased exposures due to netting.

Appendix B

VaR

The most popular metric in the banking industry to measure a portfolio's exposure to credit default risk and market risk is the so-called Value at Risk (VaR). VaR is defined as an estimate to a given degree of confidence of the amount of money that is at risk in a portfolio over a given time horizon (Wilmott, 2000). It is customary to calculate VaR at the 95% level of confidence.

Suppose that tomorrow's price P for a portfolio is a random variable. This price depends upon a set of n random variables, specifically tomorrow's values $\{V_i\}_{i=1}^n$ for a set of n portfolio "risk factors". Depending upon the portfolio, the risk factors might be variables such as exchange rates, interest rates, commodity prices or equity prices. The portfolio contains m contracts whose values tomorrow are denoted $\{C_i\}_{i=1}^m$. These contract values are each a function $C_k(V_1, \dots, V_n)$ of the risk factors. The "portfolio price function" describes P as a function of the risk factors, $P(V_1, \dots, V_n) = \sum_k C_k(V_1, \dots, V_n)$. The portfolio's one-day profit or loss is denoted by ΔP and is simply P minus today's value for the portfolio. The portfolio's VaR can thus also be defined as the bound on a 95% confidence interval for ΔP and this can be expressed as the solution to the following integral,

$$95\% = \int_{-VaR}^{\infty} p d\Delta P, \tag{B.1}$$

where p is the probability density function for ΔP (Holton, 1995).

In Equation B.1 we are not actually solving for the value of the integral. We know the value must be 95%. Instead, we are solving for the value VaR that makes it 95%. Depending on

the type of distribution, if no closed form solution exists for Equation B.1, one would typically consider numerical methods of integration. Examples of these methods include quadrature and MC simulation. In doing so, we face a problem called “the curse of dimensionality”. This arises because although Equation B.1 is presented as a one-dimensional integral, it is in fact an n -dimensional integral, as both p and ΔP are functions of the n risk factors $\{V_i\}_{i=1}^n$. Since a bank’s portfolio assets are not normally distributed, but skewed to the left, as shown in Figure B.1 (Ieda *et. al.*, 2000), the exercise of determining VaR becomes more complex.

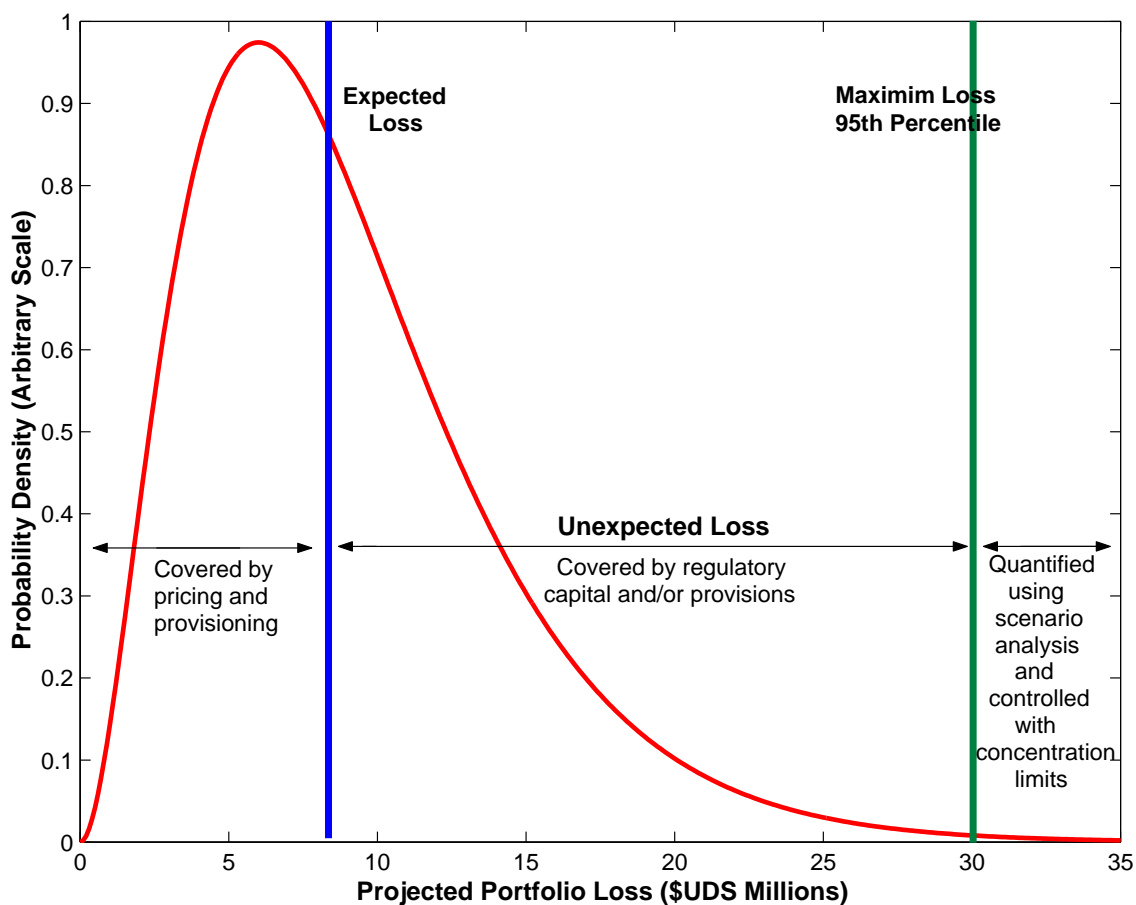


Figure B.1: Conceptual Diagram of the Density Function of a Typical Bank’s Loss Distribution.

Figure B.1 shows how a typical bank’s portfolio loss distribution is skewed to the right, indicating that the bank is more risk averse by avoiding as many high credit risk assets as possible. The lognormal is an example of such a distribution. The simulated distributions that VaR will be calculated for will be best researched by the adoption of a netting model. Parameters, constants

and variables can then be fixed and set according to one's netting research objectives.

Appendix C

Further Quasi-Random Number Details

Rosen (2000) and separately Dembo *et. al.* (2000) have applied quasi-random numbers for the MC simulation of credit risk. Quasi-random numbers are generated by constructing numbers that are as far apart or ‘maximally self avoiding’ as possible. Whilst quasi-random numbers were originally developed for numerical integration where the object one is measuring is unknown or too complex to model, they have been shown to give superior results in certain financial applications (Lemieux and L’Ecuyer, 2001). Consequently, there are many advantages and disadvantages in using MC versus Quasi-MC Simulation, and these are summarised in Table C.1.

Table C.1: The Advantages and Disadvantages of Monte Carlo and Quasi-Monte Carlo Simulation (Algorithmics, 2000).

SIMULATION APPROACH	MONTE CARLO	QUASI-MONTE CARLO
ADVANTAGES	<ul style="list-style-type: none"> • Generally applicable to all problems • Rate of convergence is independent of the dimensionality of the risk factor space • Widely used and their properties are well known 	<ul style="list-style-type: none"> • Thoroughly tested and validated • Based on sampling techniques that generate points evenly within the region and avoid clustering
DISADVANTAGES	<ul style="list-style-type: none"> • Pseudo-random number generators used tend to generate clustering about points • Do not explicitly exploit particular features of the problem • Rate of convergence is slow 	<ul style="list-style-type: none"> • Lack of generality when compared to Monte Carlo methods means that their effectiveness may be largely dependent on the problem, and extensive testing is required • Do not yield probabilistic errors or ‘<i>a priori</i>’ bounds on VaR estimates • Rate of convergence depends on the dimensionality of the risk space • May be inefficient for problems of very large dimensions

Quasi-Random Numbers are part of a more general collection of numbers called Low Discrepancy Sequences (LDS). Examples of these sequences together with their year of first publication include; Halton Sequences (1960), Sobol Sequences (1967), Faure Sequences (1982), Haselgrove Sequences (1961), and Hammersley Sequences (1961).

The Halton sequences are by far the simplest of these quasi-random numbers. From a high level overview perspective, they seem to behave better than the pseudo-random numbers, as seen by comparing the (x_n, x_{n+1}) plots in Figure C.1 below.

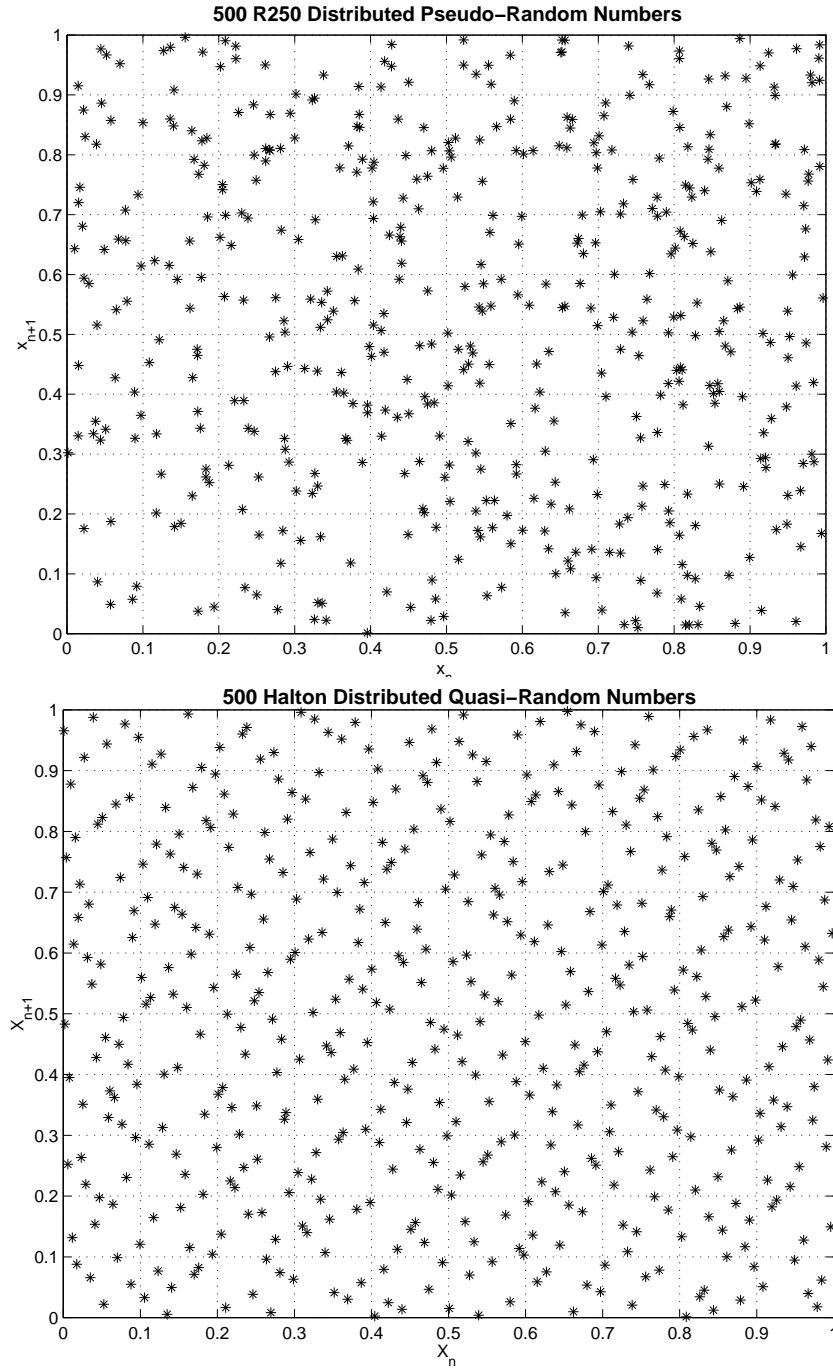


Figure C.1: 500 MATLAB (R250) Pseudo-Random and 500 Halton Quasi-Random Numbers in a (x_n, x_{n+1}) Plot or Marsaglia Plot

For every plot pairs point (x_n, x_{n+1}) in Figure C.1, x_n was determined using a Halton sequence of base 2, and x_{n+1} was determined using a Halton sequence of base 3. In general, x_{n+i} in (x_n, \dots, x_{n+i}) is generated using a Halton sequence of base $i + 2$. Typically, when one analyses these numbers in higher dimensions, say in a (x_n, x_{n+1}, x_{n+2}) Plot or a 3-Dimensional Marsaglia Plot, one still finds that they do not exhibit excessive clustering. A particular implementation of a Halton Sequence algorithm was trialled for this research, and shows excessive banding along planes, as in Figure C.2.

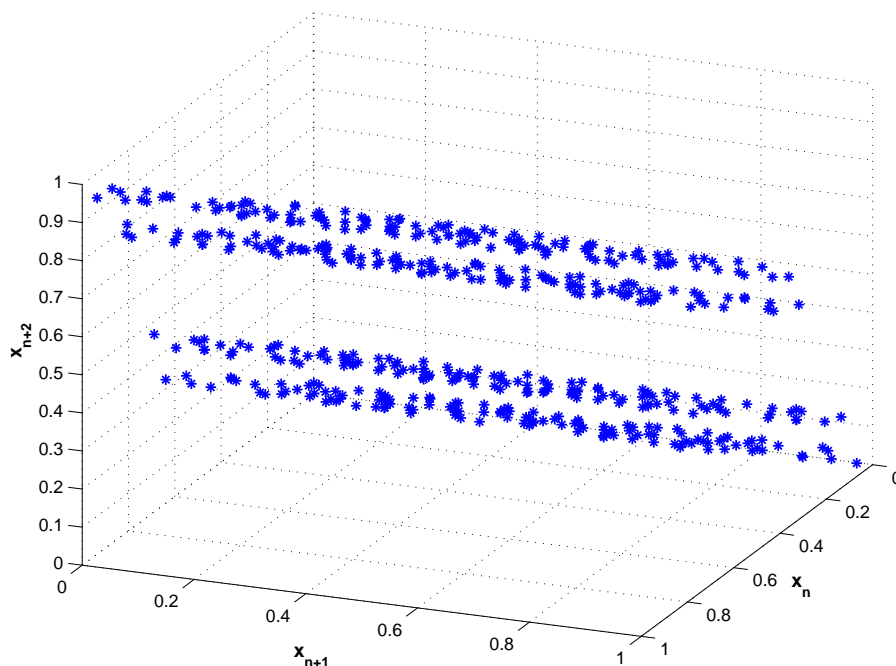


Figure C.2: 500 Halton Quasi-Random Numbers of Figure C.1 in a (x_n, x_{n+1}, x_{n+2}) Plot

For this reason of banding and due to the scope restrictions of this thesis, it was decided not to pursue quasi-random numbers any further. However, in a larger scale of research, an appropriate set of quasi-random numbers would be found that supports the work of Rosen (2000), i.e. that for certain financial simulations, quasi-random numbers produce more accurate results than using pseudo-random numbers.